



North Carolina Department of Public Instruction

## **INSTRUCTIONAL SUPPORT TOOLS**

FOR ACHIEVING NEW STANDARDS

### ***K Grade Mathematics • Unpacked Content***

For the new Common Core standards that will be effective in all North Carolina schools in the 2012-13.

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers. This document was written by the NCDPI Mathematics Consultants with the collaboration of many educators from across the state.

#### **What is the purpose of this document?**

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the CCSS.

#### **What is in the document?**

Descriptions of what each standard means a student will know, understand and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

#### **How do I send Feedback?**

We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at [denise.schulz@dpi.nc.gov](mailto:denise.schulz@dpi.nc.gov) or [kitty.rutherford@dpi.nc.gov](mailto:kitty.rutherford@dpi.nc.gov) and we will use your input to refine our unpacking of the standards. Thank You!

#### **Just want the standards alone?**

You can find the standards alone at <http://corestandards.org/the-standards>

## Standards for Mathematical Practice in Kindergarten

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Practice	Explanation and Example
<b>1. Make Sense and Persevere in Solving Problems.</b>	Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). Using both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, kindergarten students begin to reason as they become more conscious of what they know and how they solve problems.
<b>2. Reason abstractly and quantitatively.</b>	Mathematically proficient students in Kindergarten begin to use numerals to represent specific amount (quantity). For example, a student may write the numeral “11” to represent an amount of objects counted, select the correct number card “17” to follow “16” on the calendar, or build a pile of counters depending on the number drawn. In addition, kindergarten students begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation (Mary has 3 bears. Juanita gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation (Mary had 5 bears. She gave some to Juanita. Now she has 3 bears. How many bears did Mary give Juanita?). Using the language developed through numerous joining and separating scenarios, kindergarten students begin to understand how symbols (+, -, =) are used to represent quantitative ideas in a written format.
<b>3. Construct viable arguments and critique the reasoning of others.</b>	In Kindergarten, mathematically proficient students begin to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Through opportunities that encourage exploration, discovery, and discussion, kindergarten students begin to learn how to express opinions, become skillful at listening to others, describe their reasoning and respond to others’ thinking and reasoning. They begin to develop the ability to reason and analyze situations as they consider questions such as, “Are you sure...?” , “Do you think that would happen all the time...?” , and “I wonder why...?”
<b>4. Model with mathematics.</b>	Mathematically proficient students in Kindergarten begin to experiment with representing real-life problem situations in multiple ways such as with numbers, words (mathematical language), drawings, objects, acting out, charts, lists, and number sentences. For example, when making toothpick designs to represent the various combinations of the number “5”, the student writes the numerals for the various parts (such as “4” and “1”) or selects a number sentence that represents that particular situation (such as $5 = 4 + 1$ )*.  *According to CCSS, “Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required”. However, please note that it is not until First Grade when “Understand the meaning of the equal sign” is an expectation (1.OA.7).

<p><b>5. Use appropriate tools strategically.</b></p>	<p>In Kindergarten, mathematically proficient students begin to explore various tools and use them to investigate mathematical concepts. Through multiple opportunities to examine materials, they experiment and use both concrete materials (e.g. 3-dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, calculators, interactive websites) to explore mathematical concepts. Based on these experiences, they become able to decide which tools may be helpful to use depending on the problem or task. For example, when solving the problem, “There are 4 dogs in the park. 3 more dogs show up in the park. How many dogs are in the park?”, students may decide to act it out using counters and a story mat; draw a picture; or use a handful of cubes.</p>
<p><b>6. Attend to precision</b></p>	<p>Mathematically proficient students in Kindergarten begin to express their ideas and reasoning using words. As their mathematical vocabulary increases due to exposure, modeling, and practice, kindergarteners become more precise in their communication, calculations, and measurements. In all types of mathematical tasks, students begin to describe their actions and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise explanations and reasoning regarding their process of finding solutions. For example, a student may use color words (such as blue, green, light blue) and descriptive words (such as small, big, rough, smooth) to accurately describe how a collection of buttons is sorted.</p>
<p><b>7. Look for and make use of structure</b></p>	<p>Mathematically proficient students in Kindergarten begin to look for patterns and structures in the number system and other areas of mathematics. For example, when searching for triangles around the room, kindergarteners begin to notice that some triangles are larger than others or come in different colors- yet they are all triangles. While exploring the part-whole relationships of a number using a number balance, students begin to realize that 5 can be broken down into sub-parts, such as 4 and 1 or 4 and 2, and still remain a total of 5.</p>
<p><b>8. Look for and express regularity in repeated reasoning.</b></p>	<p>In Kindergarten, mathematically proficient students begin to notice repetitive actions in geometry, counting, comparing, etc. For example, a kindergartener may notice that as the number of sides increase on a shape, a new shape is created (triangle has 3 sides, a rectangle has 4 sides, a pentagon has 5 sides, a hexagon has 6 sides). When counting out loud to 100, kindergartners may recognize the pattern 1-9 being repeated for each decade (e.g., Seventy-ONE, Seventy-TWO, Seventy-THREE... Eighty-ONE, Eighty-TWO, Eighty-THREE...). When joining one more cube to a pile, the child may realize that the new amount is the next number in the count sequence.</p>

## Kindergarten Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for Kindergarten can be found on page 9 in the *Common Core State Standards for Mathematics*.

**1. Representing, relating, and operating on whole numbers, initially with sets of objects.**

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as  $5 + 2 = 7$  and  $7 - 2 = 5$ . (*Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.*) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

**2. Describing shapes and space.**

Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

# Counting and Cardinality

K.CC

## Common Core Standard and Cluster

### Know number names and the count sequence.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: Introduce written number words zero, one, two...ten (students are not responsible for being able to read these words, but they should be introduced); Know digits and orally count to one hundred

#### Common Core Standard

#### Unpacking

What do these standards mean a child will know and be able to do?

**K.CC.1** Count to 100 by ones and by tens.

Students rote count by starting at one and counting to 100. When counting by ones, students need to understand that the next number in the sequence is one more. When students count by tens they are only expected to master counting on the decade (0, 10, 20, 30, 40 ...). When counting by tens, students need to understand that the next number in the sequence is “ten more” (or one more group of ten). This objective does not require recognition of numerals. It is focused on the rote number sequence.

**K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Students begin a rote forward counting sequence from a number other than 1. Thus, given the number 4, the student would count, “4, 5, 6, 7 ...” This objective does not require recognition of numerals. It is focused on the rote number sequence 0-100.

**K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Students write the numerals 0-20 and use the written numerals 0-20 to represent the amount within a set. For example, if the student has counted 9 objects, then the written numeral “9” is recorded. Students can record the quantity of a set by selecting a number card/tile (numeral recognition) or writing the numeral. Students can also create a set of objects based on the numeral presented. For example, if a student picks up the number card “13”, the student then creates a pile of 13 counters. While children may experiment with writing numbers beyond 20, this standard places emphasis on numbers 0-20.

Due to varied development of fine motor and visual development, reversal of numerals is anticipated. While reversals should be pointed out to students and correct formation modeled in instruction, the emphasis of this standard is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself.

## Common Core Cluster

### Count to tell the number of objects.

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems such as counting objects in a set, counting out a given number of objects, and comparing sets or numerals.

When learning to count, it is important for kindergarten students to connect the collection of items (4 cubes), the number word (“four”), and the numeral (4), ultimately creating a mental picture of a number. If students simply rote-count a collection of objects without connecting these three components together, they “engage in a meaningless exercise of calling numbers that are one more than the last.” (Midget, 2012) Subitizing, the ability to “instantly see how many” (Clements, 1999), helps students form a mental picture of a number. When students recognize a small collection of objects (e.g., 2 sets of two dots) as one group (e.g., four) – they are beginning to unitize. This ability to see a set of objects as a group is an important step toward being able to see smaller groups of objects within a total collection- which is necessary to decompose number. Materials such as dot cards, dice, and dominoes provide students opportunities to see a variety of patterned arrangements to develop instant recognition of small amounts.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: number words (**zero - one hundred**), **how many, count on**

### Common Core Standard

### Unpacking

What do these standards mean a child will know and be able to do?

**K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality.

Students count a set of objects and see sets and numerals in relationship to one another. These connections are higher-level skills that require students to analyze, reason about, and explain relationships between numbers and sets of objects. The expectation is that students are comfortable with these skills with the numbers 1-20 by the end of Kindergarten.

**a.** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Students implement correct counting procedures by pointing to one object at a time (one-to-one correspondence), using one counting word for every object (synchrony/ one-to-one tagging), while keeping track of objects that have and have not been counted. This is the foundation of counting.

**b.** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

Students answer the question “How many are there?” by counting objects in a set and understanding that the last number stated when counting a set (...8, 9, **10**) represents the total amount of objects: “There are **10** bears in this pile.” (cardinality). Since an important goal for children is to count with meaning, it is important to have children answer the question, “How many do you have?” after they count. Often times, children who have not developed cardinality will count the amount again, not realizing that the **10** they stated means 10 objects in all.

Young children believe what they see. Therefore, they may believe that a pile of cubes that they counted may be more if spread apart in a line. As children move towards the developmental milestone of conservation of number, they develop the understanding that the number of objects does not change when the objects are moved, rearranged, or hidden. Children need many different experiences with counting objects, as well as maturation, before they can reach this developmental milestone.



<p>c. Understand that each successive number name refers to a quantity that is one larger.</p>	<p>Another important milestone in counting is inclusion (aka hierarchal inclusion). Inclusion is based on the understanding that numbers build by exactly one each time and that they nest within each other by this amount. For example, a set of three objects is nested within a set of 4 objects; within this same set of 4 objects is also a set of two objects and a set of one. Using this understanding, if a student has four objects and wants to have 5 objects, the student is able to add one more- knowing that four is within, or a sub-part of, 5 (rather than removing all 4 objects and starting over to make a new set of 5). This concept is critical for the later development of part/whole relationships.</p> <p>Students are asked to understand this concept with and without (0-20) objects. For example, after counting a set of 8 objects, students answer the question, “How many would there be if we added one more object?”; and answer a similar question when not using objects, by asking hypothetically, “What if we have 5 cubes and added one more. How many cubes would there be then?”</p>
<p><b>K.CC.5</b> Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</p>	<p>In order to answer “how many?” students need to keep track of objects when counting. Keeping track is a method of counting that is used to count each item once and only once when determining how many. After numerous experiences with counting objects, along with the developmental understanding that a group of objects counted multiple times will remain the same amount, students recognize the need for keeping track in order to accurately determine “how many”. Depending on the amount of objects to be counted, and the students’ confidence with counting a set of objects, students may move the objects as they count each, point to each object as counted, look without touching when counting, or use a combination of these strategies. It is important that children develop a strategy that makes sense to them based on the realization that keeping track is important in order to get an accurate count, as opposed to following a rule, such as “Line them all up before you count”, in order to get the right answer.</p> <p>As children learn to count accurately, they may count a set correctly one time, but not another. Other times they may be able to keep track up to a certain amount, but then lose track from then on. Some arrangements, such as a line or rectangular array, are easier for them to get the correct answer but may limit their flexibility with developing meaningful tracking strategies, so providing multiple arrangements help children learn how to keep track. Since scattered arrangements are the most challenging for students, this standard specifies that students only count up to 10 objects in a scattered arrangement and count up to 20 objects in a line, rectangular array, or circle.</p>

## Common Core Cluster

### Compare numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **greater, more, less, fewer, equal, same amount, compare**

#### Common Core Standard

#### Unpacking

What do these standards mean a child will know and be able to do?

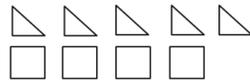
**K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>

<sup>1</sup>Include groups with up to ten objects.

Students use their counting ability to compare sets of objects (0-10). They may use matching strategies (Student 1), counting strategies (Student 2) or equal shares (Student 3) to determine whether one group is greater than, less than, or equal to the number of objects in another group.

Student 1

I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares.



Student 2

I counted the squares and I got 4. Then I counted the triangles and got 5. Since 5 is bigger than 4, there are more triangles than squares.

Student 3

I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

**K.CC.7** Compare two numbers between 1 and 10 presented as written numerals.

Students apply their understanding of numerals 1-10 to compare one numeral from another. Thus, looking at the numerals 8 and 10, a student is able to recognize that the numeral 10 represents a larger amount than the numeral 8. Students need ample experiences with actual sets of objects (K.CC.3 and K.CC.6) before completing this standard with only numerals.

# Operations and Algebraic Thinking

K.0A

## Common Core Standard and Cluster

### Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although CCSS-M states, “Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required”, please note that it is not until First Grade when “Understand the meaning of the equal sign” is an expectation (1.OA.7).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **join, add, putting together, taking apart, taking from, separate, subtract, and, same amount as, equal, less, more, total, count on**

**Note on vocabulary:** The term “total” is used here instead of the term “sum.” “Sum” sound the same as “some,” but has the opposite meaning. “Some” is used to describe problem situations with one or both addends unknown, so it is better in the earlier grades to use “total” rather than “sum.” Formal vocabulary for subtraction (“minuend” and “subtrahend”) is not needed for Kindergarten, Grade 1, and Grade 2, and may inhibit students seeing and discussing relationships between addition and subtraction. At these grades, the terms “total” and “addend” are sufficient for classroom discussion.

(*Progressions for the CCSSM*; CCSS Writing Team, May 2011, page 8)

### Common Core Standard

### Unpacking

What do these standards mean a child will know and be able to do?

**K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings<sup>2</sup>, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

<sup>2</sup>Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Students demonstrate the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations in various ways. This objective is focused on understanding the concept of addition and subtraction, rather than reading and solving addition and subtraction number sentences (equations).

Common Core State Standards for Mathematics states, “Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.” Please note that it is not until First Grade when “Understand the meaning of the equal sign” is an expectation (1.OA.7).

Therefore, before introducing symbols (+, -, =) and equations, kindergarteners require numerous experiences using joining (addition) and separating (subtraction) vocabulary in order to attach meaning to the various symbols. For example, when explaining a solution, kindergartens may state, “Three *and* two *is the same amount as* 5.” While the meaning of the equal sign is not introduced as a standard until First Grade, if equations are going to be modeled and used in Kindergarten, students must connect the symbol (=) with its meaning (is the same amount/quantity as).

**K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Kindergarten students solve four types of problems within 10: Result Unknown/Add To; Result Unknown/Take From; Total Unknown/Put Together-Take Apart; and Both Addends Unknown/Put Together-Take Apart (See **Table 1** at end of document for examples of all problem types). Kindergarteners use counting to solve the four problem types by acting out the situation and/or with objects, fingers, and drawings.

<b>Add To Result Unknown</b>	<b>Take From Result Unknown</b>	<b>Put Together/Take Apart Total Unknown</b>	<b>Put Together/Take Apart Both Addends Unknown</b>
<p>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</p> $2 + 3 = ?$	<p>Five apples were on the table. I ate two apples. How many apples are on the table now?</p> $5 - 2 = ?$	<p>Three red apples and two green apples are on the table. How many apples are on the table?</p> $3 + 2 = ?$	<p>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?</p> $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

Example: **Nine grapes were in the bowl. I ate 3 grapes. How many grapes are in the bowl now?**

**Student:** I got 9 “grapes” and put them in my bowl. Then, I took 3 grapes out of the bowl. I counted the grapes still left in the bowl... 1, 2, 3, 4, 4, 5, 6. Six. There are 6 grapes in the bowl.

Example: **Six crayons are in the box. Some are red and some are blue. How many crayons can be red, and how many crayons can be blue?**

**(Possible solution) Student:** I have 6 crayons. I moved these two over and pretended they were red. Then, I counted the “blue” ones... 1, 2, 3, 4. Four. There are 4 blue crayons.



**K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).

Students develop an understanding of part-whole relationships as they recognize that a set of objects (5) can be broken into smaller sub-sets (3 and 2) and still remain the total amount (5). In addition, this objective asks students to realize that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decompose), students use the understanding that a smaller set of objects exists within that larger set (inclusion).

Example: **“Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas.**

Students could draw pictures of:

4 blue and 1 red button      3 blue and 2 red buttons      2 blue and 3 red buttons      1 blue and 4 red buttons

In Kindergarten, students need ample experiences breaking apart numbers and using the vocabulary “and” & “same amount as” before symbols (+, =) and equations ( $5 = 3 + 2$ ) are introduced. If equations are used, a mathematical representation (picture, objects) needs to be present as well.

**K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Students build upon the understanding that a number (less than or equal to 10) can be decomposed into parts (K.OA.3) to find a missing part of 10. Through numerous concrete experiences, kindergarteners model the various sub-parts of ten and find the missing part of 10.

Example:

When working with 2-color beans, a student determines that 4 more beans are needed to make a total of 10.



“I have 6 beans. I need 4 more beans to have 10 in all.”

In addition, kindergarteners use various materials to solve tasks that involve decomposing and composing 10.

Example:

**“A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?”**

**Student A:**

*Using a Ten-Frame*

“I used a ten frame for the case. Then, I put on 6 counters for juice still in the case. There’s no juice in these 4 spaces. So, 4 are missing.”

**Student B:**

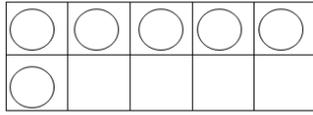
*Think Addition*

“I counted out 10 counters because I knew there needed to be ten. I pushed these 6 over here because they were in the container. These are left over. So there’s 4 missing.”

**Student C:**

*Fluently add/subtract*

“I know that it’s 4 because 6 and 4 is the same amount as 10.”



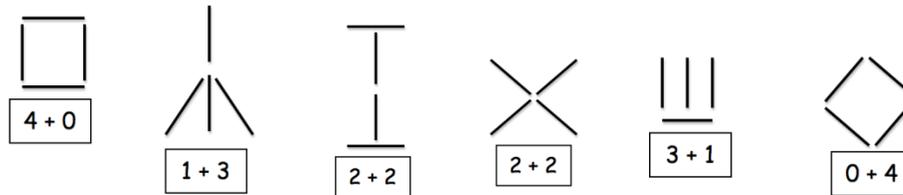
**K.OA.5** Fluently add and subtract within 5.

Students are fluent when they display accuracy (correct answer), efficiency (a reasonable amount of steps in about 3-5 seconds\* without resorting to counting), and flexibility (using strategies such as the distributive property).

Students develop fluency by understanding and internalizing the relationships that exist between and among numbers. Oftentimes, when children think of each “fact” as an individual item that does not relate to any other “fact”, they are attempting to memorize separate bits of information that can be easily forgotten. Instead, in order to fluently add and subtract, children must first be able to see sub-parts within a number (inclusion, K.CC.4.c).

Once they have reached this milestone, children need repeated experiences with many different types of concrete materials (such as cubes, chips, and buttons) over an extended amount of time in order to recognize that there are only particular sub-parts for each number. Therefore, children will realize that if 3 and 2 is a combination of 5, then 3 and 2 cannot be a combination of 6.

For example, after making various arrangements with toothpicks, students learn that only a certain number of sub-parts exist within the number 4:



Then, after numerous opportunities to explore, represent and discuss “4”, a student becomes able to fluently answer problems such as, “One bird was on the tree. Three more birds came. How many are on the tree now?”; and “There was one bird on the tree. Some more came. There are now 4 birds on the tree. How many birds came?”.

Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency.\*\* Rather, numerous experiences with breaking apart actual sets of objects and developing relationships between numbers help children internalize parts of number and develop efficient strategies for fact retrieval.

\* Van de Walle & Lovin (2006). Teaching student centered mathematics K-3 (p.94). Boston: Pearson.

## Number and Operations in Base Ten

**K.NBT**

### Common Core Standard and Cluster

#### Work with numbers 11–19 to gain foundations for place value.

Rather than unitizing a ten (recognizing that a set of 10 objects is a unit called a “ten”), which is a standard for First Grade (1.NBT.1a), kindergarteners keep each count as a single unit as they explore a set of 10 objects and leftovers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: number words (**one, two... thirteen, fourteen, ... nineteen**), **leftovers, ones**

#### Common Core Standard

**K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ )\*; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

\* Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.

#### Unpacking

What do these standards mean a child will know and be able to do?

Students explore numbers 11-19 using representations, such as manipulatives or drawings. Keeping each count as a single unit, kindergarteners use 10 objects to represent “10” rather than creating a unit called a ten (unitizing) as indicated in the First Grade CCSS standard 1.NBT.1a: 10 can be thought of as a bundle of ten ones — called a “ten.”

Example:

**Teacher:** “I have some chips here. Do you think they will fit on our ten frame? Why? Why Not?”

**Students:** Share thoughts with one another.

**Teacher:** “Use your ten frame to investigate.”

**Students:** “Look. There’s too many to fit on the ten frame. Only ten chips will fit on it.”

**Teacher:** “So you have some leftovers?”

**Students:** “Yes. I’ll put them over here next to the ten frame.”

**Teacher:** “So, how many do you have in all?”

**Student A:** “One, two, three, four, five... ten, eleven, twelve, thirteen, fourteen. I have fourteen. Ten fit on and four didn’t.”

**Student B:** Pointing to the ten frame, “See them- that’s 10... 11, 12, 13, 14. There’s fourteen.”

**Teacher:** Use your recording sheet (or number sentence cards) to show what you found out.

Student Recording Sheets Example:

ALL	On	Off
14	10	4

$14 = 10 + 4$

## Measurement and Data

**K.MD**

### Common Core Standard and Cluster

#### Describe and compare measurable attributes.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **length, weight, heavy(ier), light(er), long(er), big(ger), small(er), more of, less of, longer, taller, shorter.**

#### Common Core Standard

**K.MD.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

#### Unpacking

What do these standards mean a child will know and be able to do?

Students describe measurable attributes of objects, such as length, weight, and size. For example, a student may describe a shoe with one attribute, “My shoe is heavy!”, or more than one attribute, “This shoe is heavy! It’s also really long.”

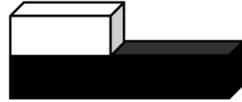
Students often initially hold undifferentiated views of measurable attributes, saying that one object is “bigger” than another whether it is longer, or greater in area, or greater in volume, and so forth. For example, two students might both claim their block building is “the biggest.” Conversations about how they are comparing- one building may be taller (greater in length) and another may have a larger base (greater in area) help students learn to discriminate and name these measureable attributes. As they discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measureable attributes of a single object. Thus, teachers listen for and extend conversations about things that are “big”, or “small,” as well as “long,” “tall,” or “high,” and name, discuss, and demonstrate with gestures the attribute being discussed.

*Progressions for the CCSSM: Geometric Measurement, The CCSS Writing Team, June 2012, page 6*

**K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference.

*For example, directly compare the heights of two children and describe one child as taller/shorter.*

Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, “The blue block is a lot longer than the white one.” Students are not comparing objects that cannot be moved and lined up next to each other.

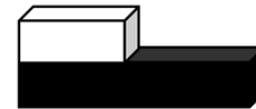


Similar to the development of the understanding that keeping track is important to obtain an accurate count, kindergarten students need ample experiences with comparing objects in order to discover the importance of lining up the ends of objects in order to have an accurate measurement.

As this concept develops, children move from the idea that “Sometimes this block is longer than this one and sometimes it’s shorter (depending on how I lay them side by side) and that’s okay.” to the understanding that “This block is always longer than this block (with each end lined up appropriately).” Since this understanding requires conservation of length, a developmental milestone for young children, kindergarteners need multiple experiences measuring a variety of items and discussing findings with one another.



“Sometimes this block is longer and sometimes it’s shorter.”



“The dark block is always longer than this block”

As students develop conservation of length, learning and using language such as “It looks longer, but it really isn’t longer” is helpful.

## Common Core Cluster

### Classify objects and count the number of objects in each category.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: color words (e.g., **blue, green, red**, etc.), descriptive words (e.g., **small, big, rough, smooth, bumpy, round, flat**, etc.), **more, less, same amount, compare, sort, category**

#### Common Core Standard

#### Unpacking

What do these standards mean a child will know and be able to do?

**K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories

Students identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to sort a collection of objects. Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.

by count.  
*(Limit category counts to be less than or equal to 10)*

Thus, like amounts are grouped together, but not necessarily ordered.

For example, **when exploring a collection of buttons:**

First, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.).

Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4).

Finally, the student organizes the groups by the quantity. “I put the purple buttons next to the green buttons because purple also had (4). Blue has 5 and orange has 3. There aren’t any other colors that have 5 or 3. So they are sitting by themselves.”

This objective helps to build a foundation for data collection in future grades as they create and analyze various graphical representations.

## Common Core Standard and Cluster

### Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

This entire cluster asks students to understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and other attributes do not (color, size, orientation). Using geometric attributes, the student identifies and describes squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Throughout the year, Kindergarten students move from informal language to describe what shapes look like (e.g., “That looks like an ice cream cone!”) to more formal mathematical language (e.g., “That is a triangle. All of its sides are the same length”).

In Kindergarten, students need ample experiences exploring various forms of the shapes (e.g., *size*: big and small; *types*: triangles, equilateral, isosceles, scalene; *orientation*: rotated slightly to the left, ‘upside down’) using geometric vocabulary to describe the different shapes.

Students in Kindergarten typically recognize figures by appearance alone, often by comparing them to a known example of a shape, such as the triangle on the left (see below). For example, students in Kindergarten typically recognize that the figure on the left as a triangle, but claim that the figure on the right is not a triangle, since it does not have a flat bottom. Thus, the properties of a figure are not recognized or known. Students typically make decisions on identifying and describing shapes based on perception, not reasoning.



Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres, flat, solid, side, corner, angle, edge, face**, positional vocabulary (e.g., **above, below, beside, in front of, behind, next to, same, different**, etc.),

Common Core Standards	Unpacking What do these standards mean a child will know and be able to do?
<p><b>K.G.1</b> Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to</i>.</p>	<p>Students locate and identify shapes in their environment. For example, a student may look at the tile pattern arrangement on the hall floor and say, “Look! I see squares! They are next to the triangle.” At first students may use informal names e.g., “balls,” “boxes,” “cans”. Eventually students refine their informal language by learning mathematical concepts and vocabulary and identify, compare, and sort shapes based on geometric attributes.*</p> <p>Students also use positional words (such as those italicized in the standard) to describe objects in the environment, developing their spatial reasoning competencies. Kindergarten students need numerous experiences identifying the location and position of actual two-and-three-dimensional objects in their classroom/school prior to describing location and position of two-and-three-dimension representations on paper.</p> <p><i>*Progressions for the CCSS in Mathematics: Geometry, The Common Core Standards Writing Team, June 2012</i></p>
<p><b>K.G.2</b> Correctly name shapes regardless of their orientations or</p>	<p>Through numerous experiences exploring and discussing shapes, students begin to understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and that other attributes do not</p>

overall size.	(color, size, orientation). As the teacher facilitates discussions about shapes (“Is it still a triangle if I turn it like this?”), children question what they “see” and begin to focus on the geometric attributes.  Kindergarten students typically do not yet recognize triangles that are turned upside down as triangles, since they don’t “look like” triangles. Students need ample experiences manipulating shapes and looking at shapes with various typical and atypical orientations. Through these experiences, students will begin to move beyond what a shape “looks like” to identifying particular geometric attributes that define a shape.
<b>K.G.3</b> Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”).	Students identify objects as flat (2 dimensional) or solid (3 dimensional). As the teacher embeds the vocabulary into students’ exploration of various shapes, students use the terms two-dimensional and three-dimensional as they discuss the properties of various shapes.

### Common Core Cluster

#### Analyze, compare, create, and compose shapes.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **compare, compose, attributes, sides, vertices/corners, vertex, two-and three-dimensional, same, different**

#### Common Core Standard

#### Unpacking

What do these standards mean a child will know and be able to do?

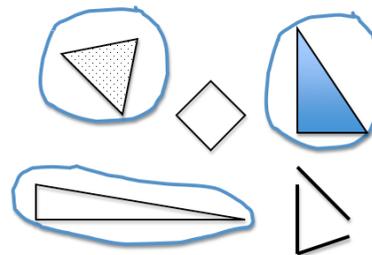
**K.G.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

Students relate one shape to another as they note similarities and differences between and among 2-D and 3-D shapes using informal language.

For example, when comparing a triangle and a square, they note that they both are closed figures, have straight sides, but the triangle has 3 sides while the square has 4. Or, when building in the Block Center, they notice that the faces on the cube are all square shapes.

Kindergarteners also distinguish between the most typical examples of a shape from obvious non-examples.

For example: When identifying the triangles from a collection of shapes, a student circles all of the triangle examples from the non-examples.



**K.G.5** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Students apply their understanding of geometric attributes of shapes in order to create given shapes. For example, students may roll a clump of play-doh into a sphere or use their finger to draw a triangle in the sand table, recalling various attributes in order to create that particular shape.

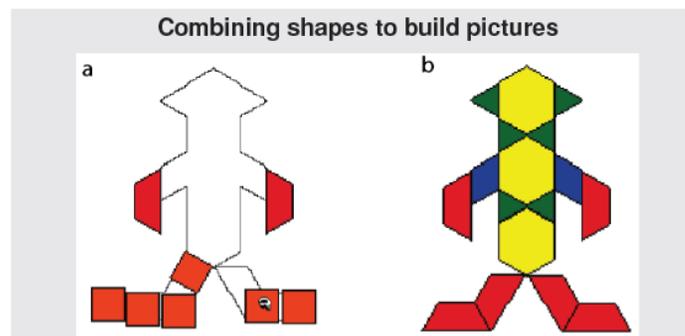
Because two-dimensional shapes are flat and three-dimensional shapes are solid, students may draw or build two-dimensional shapes and only build three-dimensional shapes. Shapes could be built using materials such as clay, toothpicks, marshmallows, gumdrops, straws, pipe cleaners, etc. Students should understand and identify two-dimensional shapes used to construct three-dimensional shapes.

**K.G.6** Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”

This standard moves beyond identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. This concept begins to develop as students move, rotate, flip, and arrange puzzle pieces to complete a puzzle. Kindergarteners use their experiences with puzzles to use simple shapes to create different shapes.

For example, when using basic shapes to create a picture, a student flips and turns triangles to make a rectangular house.

Students also combine shapes to build pictures. They first use trial and error (part a) and gradually consider components (part b)\*.



*\*Progressions for the Common Core State Standards in Mathematics: Geometry, The Common Core Standards Writing Team, June 2012, page 7*

# Glossary

**Table 1 Common addition and subtraction situations<sup>1</sup>**

	<b>Result Unknown</b>	<b>Change Unknown</b>	<b>Start Unknown</b>
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$  <b>(K)</b>	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$  <b>(1<sup>st</sup>)</b>	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$  <b>One-Step Problem (2<sup>nd</sup>)</b>
	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$  <b>(K)</b>	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$  <b>(1<sup>st</sup>)</b>	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$  <b>One-Step Problem (2<sup>nd</sup>)</b>
<b>Put Together/ Take Apart<sup>3</sup></b>	<b>Total Unknown</b>	<b>Addend Unknown</b>	<b>Both Addends Unknown<sup>2</sup></b>
	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$  <b>(K)</b>	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$  <b>(1<sup>st</sup>)</b>	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$  <b>(K)</b>
<b>Compare<sup>4</sup></b>	<b>Difference Unknown</b>	<b>Bigger Unknown</b>	<b>Smaller Unknown</b>
	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?  <b>(1<sup>st</sup>)</b>	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?  <b>One-Step Problem (1<sup>st</sup>)</b>	(Version with “more”): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have?  $5 - 3 = ? \quad ? + 3 = 5$  <b>One-Step Problem (2<sup>nd</sup>)</b>
	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$  <b>(1<sup>st</sup>)</b>	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$  <b>One-Step Problem (2<sup>nd</sup>)</b>	(Version with “fewer”): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?  <b>One-Step Problem (1<sup>st</sup>)</b>

**K:** Problem types to be mastered by the end of the Kindergarten year.

**1st:** Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

**2nd:** Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.



1Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

2These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

3Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

4For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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