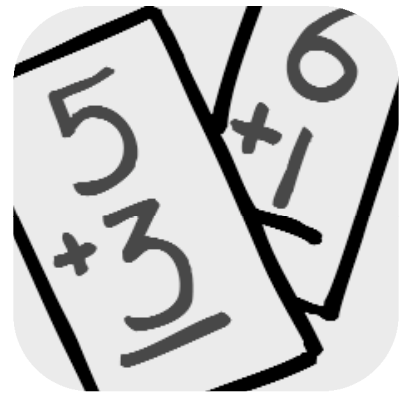


# 2003 MATHEMATICS



Standard Course of Study and  
Grade Level Competencies

**K-12**

- **K-8 Implementation 2004-05**
- **9-12 Implementation 2005-06**





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# ACKNOWLEDGMENTS

The Department of Public Instruction gratefully acknowledges the cooperation and assistance received from individuals and groups throughout the State in this current revision process. Without such cooperation, the revision of the *North Carolina Standard Course of Study* would not have been possible.

We wish to express special thanks to:

- the Office of Instructional Services for providing the leadership and vision that guided the development of this document,
- the many local educators, parents, and business people who participated in the current revision process by serving on curriculum committees and responding to draft documents,
- faculty from the institutions of higher education who advised the staff and assisted in the revision of the curriculum, and
- the Department of Public Instruction staff who carried the primary responsibility for revising and editing the *Standard Course of Study*.

The current revision process involved on some level the entire mathematics education community, and its end product is a mathematics curriculum of which North Carolina can be proud.

# PREFACE

## **Intent**

The intent of the *North Carolina Mathematics Standard Course of Study* is to provide a set of mathematical competencies for each grade and high school course to ensure rigorous student academic performance standards that are uniform across the state. It is not meant to be an instructional manual. It does not provide strategies for teaching or lesson plans.

Teachers will find NCDPI-developed support documents, such as the *Strategies for Instruction in Mathematics*, more useful in lesson planning and design. These documents will provide more detailed recommendations and support for teaching and assessing the intended curriculum.

The *North Carolina Mathematics Standard Course of Study* clearly defines a curriculum supporting the ABC's school reform effort as well as the North Carolina Testing Program. These revisions maintain a forward focus, looking at what students will need to know and be able to do to be successful and contributing citizens in our state and nation in the years ahead.

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## **Revisions**

North Carolina has had a *Standard Course of Study* since 1898. The Basic Education Program was enacted into law in 1985 and called for "a set of competencies by grade level, for each curriculum area." In 1997 the Excellent Schools Act included the following:

The State Board of Education shall develop a plan to create rigorous student academic performance standards for kindergarten through eighth grade and student academic performance standards for courses in grades 9-12. The performance standards shall align, whenever possible, with the student academic performance standards developed for the National Assessment of Educational Progress (NAEP).

The *North Carolina Mathematics Standard Course of Study* was last revised in 1998. Advisory committee meetings generated discussions

centered on initiatives in mathematics education developed or published since 1998. The review included results from the Third International Mathematics and Science Study (TIMSS), Principles and Standards of School Mathematics (NCTM 2000), and the National Assessment of Educational Progress Mathematics Framework for 2005 (NAEP).

The current revisions continue to build upon the work of the North Carolina Mathematics Framework that is “based on a philosophy of the teaching and learning of mathematics that is consistent with the current research, exemplary practices, and national standards.” The primary goal of this document is to provide content requirements that lead students to attain proficiency in mathematics. The objectives set clear, concise, and measurable expectations for all students. This *North Carolina Mathematics Standard Course of Study* provides expectations that students demonstrate competence in conceptual understanding, computing, applying, and reasoning. Classroom activities should encourage students to explore, conjecture, reason logically, and use a variety of mathematical methods effectively and efficiently to solve problems.

The revisions in content for this document have been developed through a series of public hearings and the efforts of parents, teachers, education officials, and representatives of business and industry. The revisions were approved by the North Carolina State Board of Education in March 2003.

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## **Program Review**

In order to create and maintain a quality program, a continuing re-evaluation of all aspects of the mathematics education program is necessary. There continues to be an urgent need to examine:

- The roles of teachers and students in classrooms;
- The content of school mathematics;
- Assessment practices;
- The preparation and professional development of teachers; and
- The level of support for mathematics education from all parts of society.

# PHILOSOPHY

North Carolina public schools have the challenge to provide all students with the mathematical knowledge, skills, and confidence they will need to compete in a technology-oriented workforce and to be informed citizens. With national standards, research in learning, and the increasing role of the federal government in education, there is an emerging consensus about the essential elements of mathematics content and instruction.

The *North Carolina Mathematics Standard Course of Study* is organized in five strands or goals for K-8: Number and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebra. (Geometry and Measurement are combined for grades 9-12.) The objectives for each goal progress in complexity at each grade level and throughout the high school courses. The curriculum has been designed around key ideas that should not be piecemealed into incidental details that address low-level skills. Success in mathematics integrates knowledge, conjecture, and facility with a variety of mathematical concepts. The goal of mathematics instruction should be to produce learners who comprehend concepts, operations, and relationships in mathematics as well as proficiency in computation and the application of those concepts.

The early grades focus on building a strong understanding of number and fluency with mathematics to solve problems. Fundamental to these skills is knowledge of number facts, the computational processes, and the appropriate use of each operation. Together with an emphasis on using mathematics to solve problems, elementary students will build a depth of understanding enabling them to apply the content in a variety of contexts.

Middle grades content will highlight rational numbers and algebraic thinking. Students will develop fluency in solving multi-step equations and modeling linear functions.

High school courses are designed to give students the skills and knowledge required for their future. Algebraic and geometric thinking and applied mathematics are essential for all students.

Fluency in mathematics is an expectation for all students. Fluency incorporates three ideas: efficiency, accuracy, and flexibility. Students can get bogged down with procedures and calculations that lead to errors. They become efficient as they develop strategies that are manageable, understandable, easily carried out, and generate results that solve problems. Students must develop an accurate knowledge of number facts and number relationships in order to reason and solve problems well. Flexibility is the product of students' successful experiences

with problems using a variety of strategies and the analysis of the strategies to determine their efficiency and accuracy.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is critical to a student's understanding and appreciation of the subject. Students need to use correctly the concepts, skills, symbols, and vocabulary identified in the standards set in this document. Students should talk about mathematics and use the language to verify solutions to mathematical problems.

Problem solving and reasoning are stressed throughout the goals at each grade and in every course. The development of problem-solving skills is a major goal of the mathematics program. Experiences in problem-solving processes should permeate instruction. Problem solving should be integrated early and continuously into each student's mathematics education. Students need a wide range of skills and strategies to use as a tool for representing and solving a variety of problems.

Mathematical modeling is an important technique used to build understanding of abstract ideas. Teachers need to expose students to physical representations that help develop understanding of abstract concepts. Early years should include work with manipulatives to help form a sense of number, and work with geometric shapes and patterns facilitates the development of spatial reasoning. In later studies, students will generate algebraic expressions, another form of modeling, which represent physical, social, or natural phenomena and help them make predictions.

One of the challenges facing education today is the development of effective mechanisms for informing teachers about this research so that they can transform the learning environment in their classrooms. Research shows that students develop mathematical competence and power by engaging in solving meaningful problems. Beginning in the earliest grade levels, students should use their own knowledge and experience, working alone, in pairs, and in small and large groups, to solve challenging tasks. They should be expected to communicate their thinking with pictures, numbers and words. Teachers should encourage students to question one another when an explanation doesn't make sense to them. This problem-centered approach to learning mathematics will enable students to take greater responsibility for their own learning, to develop essential communication and decision-making skills, and to understand the fundamental concepts of mathematics, all of which will be critically important to them.

# PURPOSE

The vision and philosophy described throughout this document are based on our goals in mathematics education for North Carolina students.

The six goals are for *all* students to develop:

- Strong mathematical problem-solving and reasoning abilities;
- A firm grounding in essential mathematical concepts and skills, including computation and estimation;
- Connections within mathematics and with other disciplines;
- The ability to use appropriate tools including technology to solve mathematical problems;
- The ability to communicate their understanding of mathematics effectively; and
- Positive attitudes and beliefs about mathematics.

These goals for our students in mathematics are the foundation for the rest of the document and guide the development of the critical areas in mathematics education.

Five components have been identified as critical for achieving the goals for our students and as making a significant impact on the quality of mathematics education. A summary of each of these is given here. They are:

- Teaching and Learning
- Content
- Assessment
- Preparation and Professional Development of Teachers
- Roles and Responsibilities

Teachers are the keys to changing the learning environment in North Carolina's classrooms. They plan classroom experiences and create a supportive environment for learning to take place. A teacher plays many roles in today's classrooms as the guide, the coach, the facilitator, and the instigator of mathematical explorations. Through their classroom practices, teachers promote students' mathematical reasoning, challenge them with rich problems through which they learn to value mathematics, and provide them with a strong foundation for further study. Most of all, teachers encourage and support their students' development of mathematical power.

The heart of mathematics is reasoning, which, together with knowledge of appropriate mathematical content, forms the basis of mathematical power. The goal of mathematics education in North Carolina is to enable all students to develop mathematical power and

confidence in their ability to do mathematics. A curriculum that focuses on significant mathematical ideas, instead of isolated topics, encompasses both concepts and skills through rich explorations, problems, and applications that enable students to develop a genuine understanding of the big ideas of mathematics.

Assessment is a process of planning, gathering evidence, interpreting evidence, and making decisions. Mathematics assessment is directly related to instruction and student thinking. Four purposes of assessment are monitoring student progress, making instructional decisions, evaluating student achievement, and evaluating a program.

Assessment has taken on a broader meaning. Beyond grading students, assessment should probe beneath right answers to discover how students think and how instruction can be improved. In this view of assessment, expected outcomes are set and the time necessary for each student to achieve the intended outcomes varies. Every student is challenged to meet a higher standard.

Effective assessment fosters the development of mathematical power. Students must be given opportunities to acquire and demonstrate understanding and depth of knowledge. The criteria for judging mathematical accomplishment must be made public and communicated clearly to students, parents, and other appropriate parties.

Teaching in a manner that cultivates mathematical power for all students is a complex and demanding process that requires intensive lifelong learning. Teachers must have not only extensive knowledge of mathematics but also deep understanding of how students learn mathematics. Appropriate content and pedagogical preparation enables teachers to design lessons and implement curriculum using suitable strategies and resources in an environment where all students have an opportunity to succeed. Teachers are role models for their students, as learners and problem solvers who value and enjoy mathematics. Programs for teachers at all stages in their career must provide them with the tools to implement the goals previously stated.

The professional development of a mathematics teacher is an ongoing process. This development occurs in three distinct phases: pre-service (undergraduate or teacher preparation); induction (the first three to five years of teaching); and in-service (continued professional growth over the span of the teacher's career). Communication and articulation efforts to link these phases are necessary for a continuum of professional growth. Although teachers need to take a major responsibility for their own professional development, it also requires leadership, resources, financial support, and advocacy at the state, district, school, classroom, and university levels.

Many different constituencies in North Carolina must work together to establish a mathematics education program that enables all students in the state to develop their full potential as powerful and creative thinkers and problem solvers.

Support for mathematics education requires that:

- Legislators and other financial partners provide funding that increases the likelihood of student achievement;
- School boards and administrators enact policies which enable teachers to provide quality instruction;
- Colleges and universities help teachers develop content knowledge and expertise in teaching;
- Parents and other citizens become partners with schools to value and nurture student efforts; and
- Educators and students work cooperatively to establish and reach high educational goals.

The education of students is our shared responsibility. All citizens of North Carolina must assume this responsibility and become active advocates for improved mathematics education.

# THE MATHEMATICS PROGRAM

The competency goals and objectives of the *Mathematics Standard Course of Study* are organized into five strands for K-8: Number and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebra. (Geometry and Measurement are combined for grades 9-12.) These strands are not meant to be a sequential guide for instruction but rather an organization of similar objectives under a common topic.

The mathematics program is designed in grade spans that parallel the developmental stages of students: grades K-2, grades 3-5, grades 6-8, and grades 9-12. The elementary program focuses on students actively engaged in the development of mathematical understanding by using manipulatives, working independently and cooperatively to solve problems, and conducting investigations and recording findings. Middle grade students expand their skills to compute with all real numbers and are challenged to apply their prior knowledge and experience in new and more difficult situations. The basic high school mathematics program includes courses from Introductory Mathematics through Advanced Placement Calculus. Additional elective courses are intended to offer opportunities that address the needs of individual schools.

<b>Elementary School Mathematics</b>	
<b>Middle School Mathematics</b>	
<b>High School Courses</b>	
• Introductory Mathematics	• Algebra 1
• Technical Mathematics 1	• Geometry
• Technical Mathematics 2	• Algebra 2
• Integrated Mathematics 1, 2, 3, 4	
• Advanced Functions and Modeling	
• Discrete Mathematics	
• Pre-Calculus	
• Advanced Placement Statistics	
• Advanced Placement Calculus	

There are several differences for the 2003 revision of the *North Carolina Mathematics Standard Course of Study*:

- Five strands for K-8.
- Development and mastery of major concepts in specific grades.
- Extension of the Integrated Mathematics sequence to a fourth year.
- Advanced Functions and Modeling, a new high school elective.

## EARLY GRADES K-2

### **Number and Operations**

In the early grades students develop number sense, reading, writing, and counting with whole numbers. Whole numbers are represented using concrete, pictorial, and symbolic representations. Students recognize different representations for whole numbers and explain why those representations are equivalent. Whole numbers are compared and ordered and students use a variety of strategies to estimate quantities and understand place value. Students develop fluency with addition and subtraction using multiple strategies.

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### **Measurement**

Students in the early grades begin to understand the concepts of measurement by using a variety of materials. As they learn about different tools for measuring, they describe, estimate, and measure length and temperature using non-standard and standard units. Students use the calendar to follow the days of the week and months of the year. Clocks are used to explore and tell time at the five-minute intervals.

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### **Geometry**

Students learn the names and basic properties of simple geometric shapes. They learn how shapes can be cut or arranged to form new shapes. Students look for the shapes in the environment, and practice drawing and using the shapes. They learn the meaning of basic directional and positional relationships.

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### **Data Analysis and Probability**

Students are introduced to the process of statistical investigation. They collect data by counting, measuring, and conducting simple surveys and experiments. They organize, describe, and display data. Simple probability experiments are conducted and the results interpreted.

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## **Algebra**

Children in the early grades learn about patterns and describe objects by their attributes. They compare, sort, and order things by one or more characteristics. Their understanding is extended by finding and creating patterns, correcting errors in patterns, and translating patterns into different forms. Students gather data pertaining to interests, family, and other things around them. They begin to understand that a number is a symbol for how much of something there is and begin to explore the use of a variable or placeholder and write open sentences to express relationships. Students begin to use patterns as a problem-solving strategy.

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## Kindergarten

### Major Concepts/Skills

- Number sense 0 - 30
- Calendar time
- Recognize basic shapes
- Create and extend patterns
- Sort and classify

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will recognize, model, and write whole numbers through 30.**

### Objectives

- 1.01 Develop number sense for whole numbers through 30.
  - a) Connect model, number word (orally), and number, using a variety of representations.
  - b) Count objects in a set.
  - c) Read and write numerals.
  - d) Compare and order sets and numbers.
  - e) Use ordinals (1st-10th).
  - f) Estimate quantities fewer than or equal to 10.
  - g) Recognize equivalence in sets and numbers 1-10.
- 1.02 Share equally (divide) between two people; explain.
- 1.03 Solve problems and share solutions to problems in small groups.

**COMPETENCY GOAL 2: The learner will explore concepts of measurement.**

**Objectives**

- 2.01 Compare attributes of two objects using appropriate vocabulary (color, weight, height, width, length, texture).
- 2.02 Recognize concepts of calendar time using appropriate vocabulary (days of the week, months of the year, seasons).

**COMPETENCY GOAL 3: The learner will explore concepts of geometry.**

**Objectives**

- 3.01 Identify, build, draw, and name triangles, rectangles, and circles; identify, build, and name spheres and cubes.
- 3.02 Compare geometric shapes (identify likenesses and differences).
- 3.03 Model and use directional and positional vocabulary.
- 3.04 Complete simple spatial visualization tasks and puzzles.

**COMPETENCY GOAL 4: The learner will collect, organize and display data.**

**Objectives**

- 4.01 Collect and organize data as a group activity.
- 4.02 Display and describe data with concrete and pictorial graphs as a group activity.

**COMPETENCY GOAL 5: The learner will model simple patterns and sort objects.**

**Objectives**

- 5.01 Sort and classify objects by one attribute.
- 5.02 Create and extend patterns with actions, words, and objects.

## Grade 1

### Major Concepts/Skills

- Number sense 0-99
- Single digit addition and subtraction
- Time
- Non-standard measurement
- Collect and display data
- Create and extend patterns

### Concepts/Skills to Maintain

- Basic geometric shapes
- Sort and classify

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will read, write, and model whole numbers through 99 and compute with whole numbers.**

### Objectives

- 1.01 Develop number sense for whole numbers through 99.
  - a) Connect the model, number word, and number using a variety of representations.
  - b) Use efficient strategies to count the number of objects in a set.
  - c) Read and write numbers.
  - d) Compare and order sets and numbers.
  - e) Build understanding of place value (ones, tens).
  - f) Estimate quantities fewer than or equal to 100.
  - g) Recognize equivalence in sets and numbers 1-99.
- 1.02 Use groupings of 2's, 5's, and 10's with models and pictures to count collections of objects.
- 1.03 Develop fluency with single-digit addition and corresponding differences using strategies such as modeling, composing and decomposing quantities, using doubles, and making tens.
- 1.04 Create, model, and solve problems that use addition, subtraction, and fair shares (between two or three).

**COMPETENCY GOAL 2: The learner will use non-standard units of measure and tell time.**

**Objectives**

- 2.01 For given objects:
  - a) Select an attribute (length, capacity, mass) to measure (use non-standard units).
  - b) Develop strategies to estimate size.
  - c) Compare, using appropriate language, with respect to the attribute selected.
- 2.02 Develop an understanding of the concept of time.
  - a) Tell time at the hour and half-hour.
  - b) Solve problems involving applications of time (clock and calendar).

**COMPETENCY GOAL 3: The learner will identify, describe, draw, and build basic geometric figures.**

**Objectives**

- 3.01 Identify, build, draw and name parallelograms, squares, trapezoids, and hexagons.
- 3.02 Identify, build, and name cylinders, cones, and rectangular prisms.
- 3.03 Compare and contrast geometric figures.
- 3.04 Solve problems involving spatial visualization.

**COMPETENCY GOAL 4: The learner will understand and use data and simple probability concepts.**

**Objectives**

- 4.01 Collect, organize, describe and display data using line plots and tallies.
- 4.02 Describe events as certain, impossible, more likely or less likely to occur.

**COMPETENCY GOAL 5: The learner will demonstrate an understanding of classification and patterning.**

**Objectives**

- 5.01 Sort and classify objects by two attributes.
- 5.02 Use Venn diagrams to illustrate similarities and differences in two sets.
- 5.03 Create and extend patterns, identify the pattern unit, and translate into other forms.

## Grade 2

### Major Concepts/Skills

- Number sense 0-999
- Place value
- Addition and subtraction of multi-digit numbers
- Length, time
- Symmetry and congruence
- Pictographs
- Probability experiments
- Number sentences
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Patterns
- Sort and classify
- Line plots, tallies

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will read, write, model, and compute with whole numbers through 999.**

### Objectives

- 1.01 Develop number sense for whole numbers through 999.
- a) Connect model, number word, and number using a variety of representations.
  - b) Read and write numbers.
  - c) Compare and order.
  - d) Rename.
  - e) Estimate.
  - f) Use a variety of models to build understanding of place value (ones, tens, hundreds).

- 1.02 Use area or region models and set models of fractions to explore part-whole relationships in contexts.
  - a) Represent fractions (halves, thirds, fourths) concretely and symbolically.
  - b) Compare fractions (halves, thirds, fourths) using models.
  - c) Make different representations of the same fraction.
  - d) Combine fractions to describe parts of a whole.
- 1.03 Create, model, and solve problems that involve addition, subtraction, equal grouping, and division into halves, thirds, and fourths (record in fraction form).
- 1.04 Develop fluency with multi-digit addition and subtraction through 999 using multiple strategies.
  - a) Strategies for adding and subtracting numbers.
  - b) Estimation of sums and differences in appropriate situations.
  - c) Relationships between operations.
- 1.05 Create and solve problems using strategies such as modeling, composing and decomposing quantities, using doubles, and making tens and hundreds.
- 1.06 Define and recognize odd and even numbers.

**COMPETENCY GOAL 2: The learner will recognize and use standard units of metric and customary measurement.**

**Objectives**

- 2.01 Estimate and measure using appropriate units.
  - a) Length (meters, centimeters, feet, inches, yards).
  - b) Temperature (Fahrenheit).
- 2.02 Tell time at the five-minute intervals.

**COMPETENCY GOAL 3: The learner will perform simple transformations.**

**Objectives**

- 3.01 Combine simple figures to create a given shape.
- 3.02 Describe the change in attributes as two- and three-dimensional figures are cut and rearranged.
- 3.03 Identify and make:
  - a) Symmetric figures.
  - b) Congruent figures.

**COMPETENCY GOAL 4: The learner will understand and use data and simple probability concepts.**

**Objectives**

- 4.01 Collect, organize, describe and display data using Venn diagrams (three sets) and pictographs where symbols represent multiple units (2's, 5's, 10's).
- 4.02 Conduct simple probability experiments; describe the results and make predictions.

**COMPETENCY GOAL 5: The learner will recognize and represent patterns and simple mathematical relationships.**

**Objectives**

- 5.01 Identify, describe, translate, and extend repeating and growing patterns.
- 5.02 Write addition and subtraction number sentences to represent a problem; use symbols to represent unknown quantities.

## INTERMEDIATE GRADES 3-5

### **Number and Operations**

Students in the intermediate grades represent whole numbers, fractions, and decimals with concrete objects, pictures, and symbols in a variety of contexts. A firm understanding and use of the place value system and various properties of numbers is developed. Students recognize equivalent rational numbers and explain the basis for the equivalence. Fractions and decimals are compared and ordered.

A variety of tools is used to model operations with whole numbers and fractions, develop and apply different methods of computing, and relate models to standard symbolic expressions and algorithms. Students learn the order of operations, explore various properties of operations, and are able to estimate reasonable answers to computations. Students become fluent operating with whole numbers.

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### **Measurement**

Students estimate and measure temperature, length, mass, and capacity in both customary and metric units. They solve problems involving perimeter of plane figures and area of rectangles and develop the basic formulas for computing these quantities.

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### **Geometry**

In the intermediate grades, students compare, describe, classify, and analyze two- and three- dimensional figures. They investigate basic geometric relationships, such as parallelism, perpendicularity, congruence, and similarity, and recognize geometric transformations. Students plot points and read graphs on a rectangular grid.

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### **Data Analysis and Probability**

Students continue working with the process of statistical investigation, as the techniques for data collection become more sophisticated. The nature and kinds of representations used include tables, bar and circle graphs, and stem-and-leaf plots. Data are described and compared using median, mode, and range. Students design experiments and list all possible outcomes and probabilities.

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## **Algebra**

Students in the intermediate grades continue to identify and describe patterns in many situations. Tools, such as calculators and computers, are used to investigate and discover patterns. Patterns are used in geometry and other mathematics to develop new concepts. Tables and graphs are made to show relationships and then students verbally describe the patterns. Patterns are used to extend student data, suggest rules for relationships, and make predictions. Students begin to use symbols to represent unknown quantities. They use the symbols in expressions and open sentences when describing relationships and solving problems. Students begin to identify, describe, and analyze situations with constant or varying rates of change, and compare them.

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## Grade 3

### Major Concepts/Skills

- Number sense 0 - 9,999
- Multiplication and division
- Non-negative rational numbers
- Capacity and mass
- Coordinate grids
- Circle graphs
- Permutations and combinations
- Growing patterns
- Variables
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Addition and subtraction of multi-digit numbers
- Length and time
- Symmetry and congruence
- Line plots, tallies, pictographs
- Venn diagrams

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will model, identify, and compute with whole numbers through 9,999.**

### Objectives

- 1.01 Develop number sense for whole numbers through 9,999.
- a) Connect model, number word, and number using a variety of representations.
  - b) Build understanding of place value (ones through thousands).
  - c) Compare and order.
- 1.02 Develop fluency with multi-digit addition and subtraction through 9,999 using:
- a) Strategies for adding and subtracting numbers.
  - b) Estimation of sums and differences in appropriate situations.
  - c) Relationships between operations.

- 1.03 Develop fluency with multiplication from  $1 \times 1$  to  $12 \times 12$  and division up to two-digit by one-digit numbers using:
  - a) Strategies for multiplying and dividing numbers.
  - b) Estimation of products and quotients in appropriate situations.
  - c) Relationships between operations.
- 1.04 Use basic properties (identity, commutative, associative, order of operations) for addition, subtraction, multiplication, and division.
- 1.05 Use area or region models and set models of fractions to explore part-whole relationships.
  - a) Represent fractions concretely and symbolically (halves, fourths, thirds, sixths, eighths).
  - b) Compare and order fractions (halves, fourths, thirds, sixths, eighths) using models and benchmark numbers (zero, one-half, one); describe comparisons.
  - c) Model and describe common equivalents, especially relationships among halves, fourths, and eighths, and thirds and sixths.
  - d) Understand that the fractional relationships that occur between zero and one also occur between every two consecutive whole numbers.
  - e) Understand and use mixed numbers and their equivalent fraction forms.
- 1.06 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will recognize and use standard units of metric and customary measurement.**

**Objectives**

- 2.01 Solve problems using measurement concepts and procedures involving:
  - a) Elapsed time.
  - b) Equivalent measures within the same measurement system.
- 2.02 Estimate and measure using appropriate units.
  - a) Capacity (cups, pints, quarts, gallons, liters).
  - b) Length (miles, kilometers)
  - c) Mass (ounces, pounds, grams, kilograms).
  - d) Temperature (Fahrenheit, Celsius).

**COMPETENCY GOAL 3: The learner will recognize and use basic geometric properties of two- and three-dimensional figures.**

**Objectives**

- 3.01 Use appropriate vocabulary to compare, describe, and classify two- and three-dimensional figures.
- 3.02 Use a rectangular coordinate system to solve problems.
  - a) Graph and identify points with whole number and/or letter coordinates.
  - b) Describe the path between given points on the plane.

**COMPETENCY GOAL 4: The learner will understand and use data and simple probability concepts.**

**Objectives**

- 4.01 Collect, organize, analyze, and display data (including circle graphs and tables) to solve problems.
- 4.02 Determine the number of permutations and combinations of up to three items.
- 4.03 Solve probability problems using permutations and combinations.

**COMPETENCY GOAL 5: The learner will recognize, determine, and represent patterns and simple mathematical relationships.**

**Objectives**

- 5.01 Describe and extend numeric and geometric patterns.
- 5.02 Extend and find missing terms of repeating and growing patterns.
- 5.03 Use symbols to represent unknown quantities in number sentences.
- 5.04 Find the value of the unknown in a number sentence.

## Grade 4

### Major Concepts/Skills

- Number sense 0.01-99,999
- Multiplication and division of multi-digit numbers
- Perimeter and area
- Transformations
- Line graphs
- Median, mode, and range
- Variables in number sentences
- Proportional reasoning
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Whole number computation
- Non-negative rational numbers
- Length, time, capacity, and mass
- Symmetry and congruence
- Coordinate grids
- Circle graphs
- Permutations and combinations

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will read, write, model, and compute with non-negative rational numbers.**

### Objectives

- 1.01 Develop number sense for rational numbers 0.01 through 99,999.
- a) Connect model, number word, and number using a variety of representations.
  - b) Build understanding of place value (hundredths through ten thousands).
  - c) Compare and order rational numbers.
  - d) Make estimates of rational numbers in appropriate situations.
- 1.02 Develop fluency with multiplication and division:
- a) Two-digit by two-digit multiplication (larger numbers with calculator).
  - b) Up to three-digit by two-digit division (larger numbers with calculator).
  - c) Strategies for multiplying and dividing numbers.
  - d) Estimation of products and quotients in appropriate situations.
  - e) Relationships between operations.

- 1.03 Solve problems using models, diagrams, and reasoning about fractions and relationships among fractions involving halves, fourths, eighths, thirds, sixths, twelfths, fifths, tenths, hundredths, and mixed numbers.
- 1.04 Develop fluency with addition and subtraction of non-negative rational numbers with like denominators, including decimal fractions through hundredths.
  - a) Develop and analyze strategies for adding and subtracting numbers.
  - b) Estimate sums and differences.
  - c) Judge the reasonableness of solutions.
- 1.05 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will understand and use perimeter and area.**

**Objectives**

- 2.01 Develop strategies to determine the area of rectangles and the perimeter of plane figures.
- 2.02 Solve problems involving perimeter of plane figures and areas of rectangles.

**COMPETENCY GOAL 3: The learner will recognize and use geometric properties and relationships.**

**Objectives**

- 3.01 Use the coordinate system to describe the location and relative position of points and draw figures in the first quadrant.
- 3.02 Describe the relative position of lines using concepts of parallelism and perpendicularity.
- 3.03 Identify, predict, and describe the results of transformations of plane figures.
  - a) Reflections.
  - b) Translations.
  - c) Rotations.

**COMPETENCY GOAL 4: The learner will understand and use graphs, probability, and data analysis.**

**Objectives**

- 4.01 Collect, organize, analyze, and display data (including line graphs and bar graphs) to solve problems.
- 4.02 Describe the distribution of data using median, range and mode.
- 4.03 Solve problems by comparing two sets of related data.
- 4.04 Design experiments and list all possible outcomes and probabilities for an event.

**COMPETENCY GOAL 5: The learner will demonstrate an understanding of mathematical relationships.**

**Objectives**

- 5.01 Identify, describe, and generalize relationships in which:
  - a) Quantities change proportionally.
  - b) Change in one quantity relates to change in a second quantity.
- 5.02 Translate among symbolic, numeric, verbal, and pictorial representations of number relationships.
- 5.03 Verify mathematical relationships using:
  - a) Models, words, and numbers.
  - b) Order of operations and the identity, commutative, associative, and distributive properties.

## Grade 5

### Major Concepts/Skills

- Number sense 0.001-999,999
- Addition and subtraction of non-negative rational numbers
- Properties of plane figures
- Bar graphs and stem-and-leaf plots
- Rates of change
- Simple equations and inequalities
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Whole number computation
- Transformations
- Perimeter and area
- Coordinate grids
- Line graphs
- Median, mode, and range

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will understand and compute with non-negative rational numbers.**

### Objectives

- 1.01 Develop number sense for rational numbers 0.001 through 999,999.
- a) Connect model, number word, and number using a variety of representations.
  - b) Build understanding of place value (thousandths through hundred thousands).
  - c) Compare and order rational numbers.
  - d) Make estimates of rational numbers in appropriate situations.
- 1.02 Develop fluency in adding and subtracting non-negative rational numbers (halves, fourths, eighths; thirds, sixths, twelfths; fifths, tenths, hundredths, thousandths; mixed numbers).
- a) Develop and analyze strategies for adding and subtracting numbers.
  - b) Estimate sums and differences.
  - c) Judge the reasonableness of solutions.

- 1.03 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will recognize and use standard units of metric and customary measurement.**

**Objectives**

- 2.01 Estimate the measure of an object in one system given the measure of that object in another system.
- 2.02 Identify, estimate, and measure the angles of plane figures using appropriate tools.

**COMPETENCY GOAL 3: The learner will understand and use properties and relationships of plane figures.**

**Objectives**

- 3.01 Identify, define, describe, and accurately represent triangles, quadrilaterals, and other polygons.
- 3.02 Make and test conjectures about polygons involving:
  - a) Sum of the measures of interior angles.
  - b) Lengths of sides and diagonals.
  - c) Parallelism and perpendicularity of sides and diagonals.
- 3.03 Classify plane figures according to types of symmetry (line, rotational).
- 3.04 Solve problems involving the properties of triangles, quadrilaterals, and other polygons.
  - a) Sum of the measures of interior angles.
  - b) Lengths of sides and diagonals.
  - c) Parallelism and perpendicularity of sides and diagonals.

**COMPETENCY GOAL 4: The learner will understand and use graphs and data analysis.**

**Objectives**

- 4.01 Collect, organize, analyze, and display data (including stem-and-leaf plots) to solve problems.
- 4.02 Compare and contrast different representations of the same data; discuss the effectiveness of each representation.

- 4.03 Solve problems with data from a single set or multiple sets of data using median, range, and mode.

**COMPETENCY GOAL 5: The learner will demonstrate an understanding of patterns, relationships, and elementary algebraic representation.**

**Objectives**

- 5.01 Describe, extend, and generalize numeric and geometric patterns using tables, graphs, words, and symbols.
- 5.02 Use algebraic expressions, patterns, and one-step equations and inequalities to solve problems.
- 5.03 Identify, describe, and analyze situations with constant or varying rates of change.

## MIDDLE GRADES 6-8

### **Number and Operations**

Students in the middle grades represent real numbers using manipulatives, pictures, number lines, and symbols in a variety of contexts. Relationships among rational numbers are explored and equivalence among fractions, decimals, and percents is recognized and explained. Students extend their understanding of place value to decimal and scientific notation and use the properties of real numbers, including zero, one, and inverses. Numerical comparisons are expressed as ratios and rates and problems are solved using ratio, proportion, and percent.

Students develop fluency in computation with rational numbers as well as with relationships among numbers, including primes, composites, factors, and multiples. They explain exponents and square and cube roots of numbers, develop facility with estimation and mental computation, and use calculators appropriately.

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### **Measurement**

In the middle grades, the study of perimeter and area is expanded to include surface area and volume of prisms and cylinders and formulas are developed for computing these quantities. Relationships among length, perimeter, area, and volume are studied. Students draw objects to scale and use scale drawings to solve problems.

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### **Geometry**

Students use the properties and relationships among geometric figures to solve problems. Congruence, similarity using ratio and proportion, and the Pythagorean theorem are studied. Students continue to study symmetries and transformations and become proficient at visualizing and recognizing transformed figures in the coordinate plane. Three-dimensional figures are drawn and built using different views.

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**Data Analysis and Probability**

Students investigate increasingly complex data sets and, with the appropriate statistical measures, solve problems involving multiple data sets. More sophisticated representations, such as histograms, box plots, and scatter plots, highlight an increased understanding of the spread and grouping of data and the relationships between variables. Students identify basic patterns and trends in tables and charts and use them to make predictions. They describe the distribution of data using measures of central tendency. Students explore extremes in data and the misuse of representations to communicate information.

With bivariate data, students learn to recognize relationships, estimate, and make predictions. In particular, linear relationships are identified and used to investigate bivariate data.

Students conduct experiments and simulations to investigate basic probability, including dependent and independent events. Experimental results are compared with theoretical probabilities and students learn that the level of agreement between the two often depends on the number of times an experiment is repeated. Students learn to make inferences and predictions based on the outcomes of their experiments and simulations.

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**Algebra**

Students in the middle grades begin to use the language of function, identifying patterns and relationships in context and expressing them algebraically. Variables are used to describe the interdependence of quantities and build an understanding of slope as the rate of change between quantities. In order to solve problems, ordered pairs of data are generated and used to identify a linear relationship between quantities graphically and algebraically. From tables and graphs students recognize nonlinear relationships and functions. Linear equations and inequalities are solved using multiple strategies, including manipulatives, tables, guess-and-test, working backwards, and algebraic methods. Students simplify algebraic expressions involving real numbers and variables and apply algebraic methods to solve a variety of problems.

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## Grade 6

### Major Concepts/Skills

- Negative rational numbers
- Percent
- Transformations in the coordinate plane
- Probability
- Equations and inequalities
- Multiplication and division of non-negative rational numbers
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Addition and subtraction of non-negative rational numbers
- Number properties
- Perimeter and area
- Median, mode, and range
- Bar graphs and leaf plots

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will understand and compute with rational numbers.**

### Objectives

- 1.01 Develop number sense for negative rational numbers.
  - a) Connect the model, number word, and number using a variety of representations, including the number line.
  - b) Compare and order.
  - c) Make estimates in appropriate situations.
- 1.02 Develop meaning for percents.
  - a) Connect the model, number word, and number using a variety of representations.
  - b) Make estimates in appropriate situations.
- 1.03 Compare and order rational numbers.

- 1.04 Develop fluency in addition, subtraction, multiplication, and division of non-negative rational numbers.
  - a) Analyze computational strategies.
  - b) Describe the effect of operations on size.
  - c) Estimate the results of computations.
  - d) Judge the reasonableness of solutions.
- 1.05 Develop fluency in the use of factors, multiples, exponential notation, and prime factorization.
- 1.06 Use exponential, scientific, and calculator notation to write very large and very small numbers.
- 1.07 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will select and use appropriate tools to measure two- and three-dimensional figures.**

**Objectives**

- 2.01 Estimate and measure length, perimeter, area, angles, weight, and mass of two- and three-dimensional figures, using appropriate tools.
- 2.02 Solve problems involving perimeter/circumference and area of plane figures.

**COMPETENCY GOAL 3: The learner will understand and use properties and relationships of geometric figures in the coordinate plane.**

**Objectives**

- 3.01 Identify and describe the intersection of figures in a plane.
- 3.02 Identify the radius, diameter, chord, center, and circumference of a circle; determine the relationships among them.
- 3.03 Transform figures in the coordinate plane and describe the transformation.
- 3.04 Solve problems involving geometric figures in the coordinate plane.

**COMPETENCY GOAL 4: The learner will understand and determine probabilities.**

**Objectives**

- 4.01 Develop fluency with counting strategies to determine the sample space for an event. Include lists, tree diagrams, frequency distribution tables, permutations, combinations, and the Fundamental Counting Principle.
- 4.02 Use a sample space to determine the probability of an event.

- 4.03 Conduct experiments involving simple and compound events.
- 4.04 Determine and compare experimental and theoretical probabilities for simple and compound events.
- 4.05 Determine and compare experimental and theoretical probabilities for independent and dependent events.
- 4.06 Design and conduct experiments or surveys to solve problems; report and analyze results.

**COMPETENCY GOAL 5: The learner will demonstrate an understanding of simple algebraic expressions.**

**Objectives**

- 5.01 Simplify algebraic expressions and verify the results using the basic properties of rational numbers.
  - a) Identity.
  - b) Commutative.
  - c) Associative.
  - d) Distributive.
  - e) Order of operations.
- 5.02 Use and evaluate algebraic expressions.
- 5.03 Solve simple (one- and two-step) equations or inequalities.
- 5.04 Use graphs, tables, and symbols to model and solve problems involving rates of change and ratios.

## Grade 7

### Major Concepts/Skills

- Computation with rational numbers
- Ratio and proportion
- Factors and multiples
- Volume and surface area
- Measures of central tendency
- Box plots and histograms
- Equations and inequalities
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Number properties
- Percent
- Transformations in the coordinate plane
- Probability

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will understand and compute with rational numbers.**

### Objectives

- 1.01 Develop and use ratios, proportions, and percents to solve problems.
- 1.02 Develop fluency in addition, subtraction, multiplication, and division of rational numbers.
  - a) Analyze computational strategies.
  - b) Describe the effect of operations on size.
  - c) Estimate the results of computations.
  - d) Judge the reasonableness of solutions.
- 1.03 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will understand and use measurement involving two- and three-dimensional figures.**

**Objectives**

- 2.01 Draw objects to scale and use scale drawings to solve problems.
- 2.02 Solve problems involving volume and surface area of cylinders, prisms, and composite shapes.

**COMPETENCY GOAL 3: The learner will understand and use properties and relationships in geometry.**

**Objectives**

- 3.01 Using three-dimensional figures:
  - a) Identify, describe, and draw from various views (top, side, front, corner).
  - b) Build from various views.
  - c) Describe cross-sectional views.
- 3.02 Identify, define, and describe similar and congruent polygons with respect to angle measures, length of sides, and proportionality of sides.
- 3.03 Use scaling and proportional reasoning to solve problems related to similar and congruent polygons.

**COMPETENCY GOAL 4: The learner will understand and use graphs and data analysis.**

**Objectives**

- 4.01 Collect, organize, analyze, and display data (including box plots and histograms) to solve problems.
- 4.02 Calculate, use, and interpret the mean, median, mode, range, frequency distribution, and inter-quartile range for a set of data.
- 4.03 Describe how the mean, median, mode, range, frequency distribution, and inter-quartile range of a set of data affect its graph.
- 4.04 Identify outliers and determine their effect on the mean, median, mode, and range of a set of data.
- 4.05 Solve problems involving two or more sets of data using appropriate statistical measures.

**COMPETENCY GOAL 5: The learner will demonstrate an understanding of linear relations and fundamental algebraic concepts.**

**Objectives**

- 5.01 Identify, analyze, and create linear relations, sequences, and functions using symbols, graphs, tables, diagrams, and written descriptions.
- 5.02 Translate among different representations of algebraic expressions, equations and inequalities.
- 5.03 Use and evaluate algebraic expressions, linear equations or inequalities to solve problems.
- 5.04 Develop fluency in the use of formulas to solve problems.

## Grade 8

### Major Concepts/Skills

- Real numbers
- Linear functions
- Pythagorean theorem, indirect measurement
- Scatterplots
- Slope
- Equations and inequalities
- Students will solve relevant and authentic problems using appropriate technology and apply these concepts as well as those developed in earlier years

### Concepts/Skills to Maintain

- Ratio, proportion, and percent
- Factors and multiples
- Box plots and histograms
- Volume and surface area

**Strands:** Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will understand and compute with real numbers.**

### Objectives

- 1.01 Develop number sense for the real numbers.
  - a) Define and use irrational numbers.
  - b) Compare and order.
  - c) Use estimates of irrational numbers in appropriate situations.
- 1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will understand and use measurement concepts.**

**Objectives**

- 2.01 Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.
- 2.02 Apply and use concepts of indirect measurement.

**COMPETENCY GOAL 3: The learner will understand and use properties and relationships in geometry.**

**Objectives**

- 3.01 Represent problem situations with geometric models.
- 3.02 Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.
- 3.03 Identify, predict, and describe dilations in the coordinate plane.

**COMPETENCY GOAL 4: The learner will understand and use graphs and data analysis.**

**Objectives**

- 4.01 Collect, organize, analyze, and display data (including scatterplots) to solve problems.
- 4.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.
- 4.03 Identify misuses of statistical and numerical data.

**COMPETENCY GOAL 5: The learner will understand and use linear relations and functions.**

**Objectives**

- 5.01 Develop an understanding of function.
  - a) Translate among verbal, tabular, graphic, and algebraic representations of functions.
  - b) Identify relations and functions as linear or nonlinear.
  - c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.
  - d) Interpret and compare properties of linear functions from tables, graphs, or equations.

- 5.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.
- 5.03 Solve problems using linear equations and inequalities; justify symbolically and graphically.
- 5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

## HIGH SCHOOL GRADES 9-12

### **Number and Operations**

Students in the secondary years extend their understanding of numbers to include the complex numbers and become proficient with advanced numeric relationships, including exponential, logarithmic, trigonometric, and matrix expressions. Students develop fluency in operating with and evaluating algebraic expressions.

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### **Geometry and Measurement**

Appropriate tools are used to create figures and identify characteristics and properties that describe relationships among shapes. Students use and apply algebraic representations to describe geometric relationships and transformations. Basic trigonometric relationships and functions are developed to solve problems. Students will move from an inductive approach to deductive methods of proof in their study of geometric figures. A variety of proof strategies is used to verify generalizations and properties of shape and connect geometry to other branches of mathematics.

Students maintain and extend measurement skills from earlier years, especially through science and technical course work. They focus on solving measurement-related problems involving concepts of precision, tolerance, error, and multiple dimensions.

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### **Data Analysis and Probability**

Students use appropriate technology to investigate, analyze, and present data. Measures of central tendency, spread, and distribution are used to describe and analyze data. Hypotheses, arguments, and conclusions are made, tested, and evaluated based on data. Using a best-fit function for bivariate data, students interpret and apply the function in the context of a problem.

Building upon study in earlier years, students use theoretical and experimental probability to model and solve increasingly complex problems.

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## **Algebra**

Students will be expected to describe and translate among graphic, algebraic, numeric, and verbal representations of relations and use those representations to solve problems. They use symbols to represent variables, parameters, and functions and extend their use of symbols to include vectors and matrices. Students use technology to assist in developing models and analytical solutions. They use appropriate terminology and notation to define function, domain, range, composition, and inverses of functions. They expand their understanding of functions to include power, polynomial, exponential, periodic, piece-wise, and recursively defined functions. They solve equations, inequalities, and systems using algebraic, tabular, numerical, and graphical methods.

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## Introductory Mathematics

Introductory Mathematics provides students a survey of preparatory topics for high school mathematics, including the foundations for high school algebra and geometry. Appropriate technology, from manipulatives to calculators, should be used regularly for instruction and assessment.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will understand and compute with real numbers.**

### Objectives

- 1.01 Develop number sense for the real numbers.
  - a) Define and use irrational numbers.
  - b) Compare and order.
  - c) Use estimates of irrational numbers in appropriate situations.
- 1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

**COMPETENCY GOAL 2: The learner will use properties and relationships in geometry and measurement concepts to solve problems.**

### Objectives

- 2.01 Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.
- 2.02 Apply and use concepts of indirect measurement.
- 2.03 Represent problem situations with geometric models.
- 2.04 Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.
- 2.05 Identify, predict, and describe dilations in the coordinate plane.

**COMPETENCY GOAL 3: The learner will understand and use graphs and data analysis.**

**Objectives**

- 3.01 Collect, organize, analyze, and display data (including scatterplots) to solve problems.
- 3.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.
- 3.03 Identify misuses of statistical and numerical data.

**COMPETENCY GOAL 4: The learner will understand and use linear relations and functions.**

**Objectives**

- 4.01 Develop an understanding of function.
  - a) Translate among verbal, tabular, graphic, and algebraic representations of functions.
  - b) Identify relations and functions as linear or nonlinear.
  - c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.
  - d) Interpret and compare properties of linear functions from tables, graphs, or equations.
- 4.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.
- 4.03 Solve problems using linear equations and inequalities; justify symbolically and graphically.
- 4.04 Solve problems using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

## Algebra 1

Algebra 1 continues the study of algebraic concepts. It includes operations with polynomials and matrices, creation and application of linear functions and relations, algebraic representations of geometric relationships, and an introduction to nonlinear functions. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Operate with the real numbers to solve problems.
- Find, identify, and interpret the slope and intercepts of a linear relation.
- Visually determine a line of best fit for a given scatterplot; explain the meaning of the line; and make predictions using the line.
- Collect, organize, analyze, and display data to solve problems.
- Apply the Pythagorean Theorem to solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with numbers and expressions to solve problems.**

### Objectives

- 1.01 Write equivalent forms of algebraic expressions to solve problems.
  - a) Apply the laws of exponents.
  - b) Operate with polynomials.
  - c) Factor polynomials.
- 1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.
- 1.03 Model and solve problems using direct variation.

**COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.**

**Objectives**

- 2.01 Find the lengths and midpoints of segments to solve problems.
- 2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.

**COMPETENCY GOAL 3: The learner will collect, organize, and interpret data with matrices and linear models to solve problems.**

**Objectives**

- 3.01 Use matrices to display and interpret data.
- 3.02 Operate (addition, subtraction, scalar multiplication) with matrices to solve problems.
- 3.03 Create linear models for sets of data to solve problems.
  - a) Interpret constants and coefficients in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

**COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.**

**Objectives**

- 4.01 Use linear functions or inequalities to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret constants and coefficients in the context of the problem.
- 4.02 Graph, factor, and evaluate quadratic functions to solve problems.
- 4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.
- 4.04 Graph and evaluate exponential functions to solve problems.

## Geometry

Geometry continues students' study of geometric concepts building upon middle school topics. Students will move from an inductive approach to deductive methods of proof in their study of two- and three-dimensional geometric figures. Reasoning skills will be emphasized and students will broaden their use of the coordinate plane. Appropriate technology, from manipulatives to calculators and graphics software, should be used regularly for instruction and assessment.

### Prerequisites

- Apply geometric properties and relationships to solve problems.
- Use formulas to solve problems.
- Define and use linear expressions to model and solve problems.
- Operate with matrices to model and solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with real numbers to solve problems.**

### Objectives

- 1.01 Use the trigonometric ratios to model and solve problems involving right triangles.
- 1.02 Use length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
- 1.03 Use length, area, and volume to model and solve problems involving probability.

**COMPETENCY GOAL 2: The learner will use geometric and algebraic properties of figures to solve problems and write proofs.**

**Objectives**

- 2.01 Use logic and deductive reasoning to draw conclusions and solve problems.
- 2.02 Apply properties, definitions, and theorems of angles and lines to solve problems and write proofs.
- 2.03 Apply properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs:
  - a) Triangles.
  - b) Quadrilaterals.
  - c) Other polygons.
  - d) Circles.
- 2.04 Develop and apply properties of solids to solve problems.

**COMPETENCY GOAL 3: The learner will transform geometric figures in the coordinate plane algebraically.**

**Objectives**

- 3.01 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.
- 3.02 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

## Algebra 2

Algebra 2 continues students' study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Emphasis should be placed on practical applications and modeling. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Operate with matrices to solve problems.
- Create linear models, for sets of data, to solve problems.
- Use linear functions and inequalities to model and solve problems.
- Use quadratic functions to model problems and solve by factoring and graphing.
- Use systems of linear equations or inequalities to model and solve problems.
- Graph and evaluate exponential functions to solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with complex numbers, matrices, and polynomials.**

### Objectives

- 1.01 Simplify and perform operations with rational exponents and logarithms (common and natural) to solve problems.
- 1.02 Define and compute with complex numbers.
- 1.03 Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.
- 1.04 Operate with matrices to model and solve problems.
- 1.05 Model and solve problems using direct, inverse, combined and joint variation.

**COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.**

**Objectives**

- 2.01 Use the composition and inverse of functions to model and solve problems; justify results.
- 2.02 Use quadratic functions and inequalities to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 2.03 Use exponential functions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.04 Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.
- 2.05 Use rational equations to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
  - c) Identify the asymptotes and intercepts graphically and algebraically.
- 2.06 Use cubic equations to model and solve problems.
  - a) Solve using tables and graphs.
  - b) Interpret constants and coefficients in the context of the problem.
- 2.07 Use equations with radical expressions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the degree, constants, and coefficients in the context of the problem.
- 2.08 Use equations and inequalities with absolute value to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 2.09 Use the equations of parabolas and circles to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.

- 2.10 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

## Technical Mathematics 1

Technical Mathematics 1 continues students' study of algebra and geometry, building upon middle school and Algebra I topics. Problem solving, measurement, special relationships in right triangles, transformations, and geometric applications of algebra are the topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Apply geometric properties and relationships to solve problems.
- Use tables, formulas and algebraic expressions to model and solve problems.
- Define and use linear functions to model and solve problems.
- Operate with matrices to model and solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Algebra

### **COMPETENCY GOAL 1: The learner will apply various strategies to solve problems.**

#### **Objective**

- 1.01 Apply various techniques and strategies to solve problems.
- a) Select or create an appropriate graphical display for a given set of data.
  - b) Identify and represent patterns using appropriate algebraic notation.
  - c) Select and apply appropriate formulas.
  - d) Choose or create appropriate representations of two- and three-dimensional figures.

### **COMPETENCY GOAL 2: The learner will measure and apply geometric concepts to solve problems.**

#### **Objectives**

- 2.01 Select and use appropriate tools to measure two- and three-dimensional figures; interpret and communicate results with regard to precision.
- 2.02 Interpret and construct maps and scale drawings to solve problems.

- 2.03 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
- 2.04 Use the trigonometric ratios to model and solve problems involving right triangles.

**COMPETENCY GOAL 3: The learner will describe the transformation of polygons in the coordinate plane algebraically.**

**Objectives**

- 3.01 Apply algebraic and trigonometric concepts to confirm properties of geometric figures in the coordinate plane.
- 3.02 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.
- 3.03 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

## Technical Mathematics 2

Technical Mathematics 2 continues students' study of algebra and geometry, building upon middle and high school topics. Geometry, functions, and statistical methods for estimation and prediction are the topics to be studied in an application-centered environment.

Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Apply geometric properties and relationships to solve problems.
- Use tables, formulas and algebraic expressions to model and solve problems.
- Identify, graph, and use exponential and quadratic functions to solve problems.
- Create linear models, for a set of data, to solve problems.

**Strands:** Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will use properties of geometric figures to solve problems.**

### Objectives

- 1.01 Apply the properties and definitions of lines and angles to model and solve problems.
- 1.02 Apply the properties and definitions of plane figures to model and solve problems:
  - a) Triangles.
  - b) Quadrilaterals.
  - c) Other polygons.
  - d) Circles.

**COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.**

**Objectives**

- 2.01 Use quadratic equations to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 2.02 Use exponential functions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.03 Create, interpret, and analyze best-fit models of linear, exponential, and quadratic functions to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.
- 2.04 Use systems of equations and inequalities to model and solve problems using tables, graphs, matrix operations, and algebraic properties; justify results.

## Advanced Functions and Modeling

Advanced Functions and Modeling provides students an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, and exponential functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

**Strands:** Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will analyze data and apply probability concepts to solve problems.**

### Objectives

- 1.01 Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions and make predictions.
- 1.02 Summarize and analyze univariate data to solve problems.
  - a) Apply and compare methods of data collection.
  - b) Apply statistical principles and methods in sample surveys.

- c) Determine measures of central tendency and spread.
  - d) Recognize, define, and use the normal distribution curve.
  - e) Interpret graphical displays of univariate data.
  - f) Compare distributions of univariate data.
- 1.03 Use theoretical and experimental probability to model and solve problems.
- a) Use addition and multiplication principles.
  - b) Calculate and apply permutations and combinations.
  - c) Create and use simulations for probability models.
  - d) Find expected values and determine fairness.
  - e) Identify and use discrete random variables to solve problems.
  - f) Apply the Binomial Theorem.

**COMPETENCY GOAL 2: The learner will use functions to solve problems.**

**Objectives**

- 2.01 Use logarithmic (common, natural) functions to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.02 Use piecewise-defined functions to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.03 Use power functions to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
  - b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
  - c) Develop and use the law of sines and the law of cosines.
- 2.05 Use recursively-defined functions to model and solve problems.
- a) Find the sum of a finite sequence.
  - b) Find the sum of an infinite sequence.
  - c) Determine whether a given series converges or diverges.
  - d) Translate between recursive and explicit representations.

## Discrete Mathematics

Discrete Mathematics introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear and exponential functions to model and solve problems.
- Operate with matrices to model and solve problems.
- Define complex numbers and perform basic operations with them.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will use matrices and graphs to model relationships and solve problems.**

### Objectives

- 1.01 Use matrices to model and solve problems.
  - a) Display and interpret data.
  - b) Write and evaluate matrix expressions to solve problems.
- 1.02 Use graph theory to model relationships and solve problems.

**COMPETENCY GOAL 2: The learner will analyze data and apply probability concepts to solve problems.**

**Objectives**

- 2.01 Describe data to solve problems.
  - a) Apply and compare methods of data collection.
  - b) Apply statistical principles and methods in sample surveys.
  - c) Determine measures of central tendency and spread.
  - d) Recognize, define, and use the normal distribution curve.
  - e) Interpret graphical displays of data.
  - f) Compare distributions of data.
- 2.02 Use theoretical and experimental probability to model and solve problems.
  - a) Use addition and multiplication principles.
  - b) Calculate and apply permutations and combinations.
  - c) Create and use simulations for probability models.
  - d) Find expected values and determine fairness.
  - e) Identify and use discrete random variables to solve problems.
  - f) Apply the Binomial Theorem.
- 2.03 Model and solve problems involving fair outcomes:
  - a) Apportionment.
  - b) Election Theory.
  - c) Voting Power.
  - d) Fair Division.

**COMPETENCY GOAL 3: The learner will describe and use recursively-defined relationships to solve problems.**

**Objective**

- 3.01 Use recursion to model and solve problems.
  - a) Find the sum of a finite sequence.
  - b) Find the sum of an infinite sequence.
  - c) Determine whether a given series converges or diverges.
  - d) Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas.
  - e) Verify an explicit definition with inductive proof.

## Pre-Calculus

Pre-Calculus provides students an honors-level study of trigonometry, advanced functions, analytic geometry, and data analysis in preparation for calculus. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will describe geometric figures in the coordinate plane algebraically.**

### Objectives

- 1.01 Transform relations in two dimensions; describe the results algebraically and geometrically.
- 1.02 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 1.03 Operate with vectors in two dimensions to model and solve problems.

**COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.**

**Objectives**

- 2.01 Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.
  - a) Solve using graphs and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 2.02 Use trigonometric and inverse trigonometric functions to model and solve problems; justify results.
  - a) Solve using graphs and algebraic properties.
  - b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
  - c) Develop and use the law of sines and the law of cosines.
- 2.03 For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.
- 2.04 Use the composition and inverse of functions to model and solve problems.
- 2.05 Use polar equations to model and solve problems.
  - a) Solve using graphs and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 2.06 Use parametric equations to model and solve problems.
- 2.07 Use recursively-defined functions to model and solve problems.
  - a) Find the sum of a finite sequence.
  - b) Find the sum of an infinite sequence.
  - c) Determine whether a given series converges or diverges.
  - d) Translate between recursive and explicit representations.
- 2.08 Explore the limit of a function graphically, numerically, and algebraically.

## Integrated Mathematics 1

Integrated Mathematics 1 provides students the opportunity to study traditional topics from algebra, geometry, probability, and statistics in a problem-centered, connected approach. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Operate with real numbers to solve problems.
- Use formulas to solve problems.
- Find, identify, and interpret the slope and intercepts of a linear relation.
- Visually determine the line of best fit for a given scatterplot; explain the meaning of the line; and make predictions using the line.
- Collect, organize, analyze, and display data to solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with numbers and expressions to solve problems.**

### Objectives

- 1.01 Write equivalent forms of algebraic expressions to solve problems.
  - a) Apply the laws of exponents.
  - b) Operate with polynomials.
  - c) Factor polynomials.
- 1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.

**COMPETENCY GOAL 2: The learner will use properties of geometric figures to solve problems.**

**Objectives**

- 2.01 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
- 2.02 Develop and apply properties of solids to solve problems.

**COMPETENCY GOAL 3: The learner will analyze data and apply probability concepts to solve problems.**

**Objectives**

- 3.01 Use graph theory to model relationships and solve problems.
- 3.02 Use theoretical and experimental probability to model and solve problems.
  - a) Use addition and multiplication principles.
  - b) Calculate and apply permutations and combinations.
  - c) Create and use simulations for probability models.
  - d) Find expected values and determine fairness.
- 3.03 Create linear and exponential models, for sets of data, to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

**COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.**

**Objectives**

- 4.01 Use linear functions or inequalities to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 4.02 Use exponential functions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 4.03 Use systems of linear equations or inequalities in two variables to model problems and solve graphically.

## Integrated Mathematics 2

Integrated Mathematics 2 continues students' study of topics from algebra, geometry, and statistics in a problem-centered, connected approach. Functions, matrix operations, and algebraic representations of geometric concepts are the principle topics of study. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Create linear and exponential models, for sets of data, to solve problems.
- Use linear expressions to model and solve problems.
- Collect, organize, analyze, and display data to solve problems.
- Apply geometric properties and relationships to solve problems.
- Apply the Pythagorean Theorem to solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with numbers, algebraic expressions, and matrices to solve problems.**

### Objectives

- 1.01 Write equivalent forms of algebraic expressions to solve problems.
- 1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.
- 1.03 Model and solve problems using direct variation.
- 1.04 Operate with matrices to model and solve problems.

**COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.**

**Objectives**

- 2.01 Find the lengths and midpoints of segments to solve problems.
- 2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.
- 2.03 Use the trigonometric ratios to model and solve problems.
- 2.04 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

**COMPETENCY GOAL 3: The learner will collect, organize, and interpret data to solve problems.**

**Objectives**

- 3.01 Describe data to solve problems.
  - a) Apply and compare methods of data collection.
  - b) Apply statistical principles and methods in sample surveys.
  - c) Determine measures of central tendency and spread.
  - d) Recognize, define, and use the normal distribution curve.
  - e) Interpret graphical displays of data.
  - f) Compare distributions of data.
- 3.02 Create and use, for sets of data, calculator-generated models of linear, exponential, and quadratic functions to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

**COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.**

**Objectives**

- 4.01 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify steps used.
- 4.02 Use quadratic functions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 4.03 Use power models to solve problems.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.

## Integrated Mathematics 3

Integrated Mathematics 3 continues students' study of topics from algebra, geometry, and statistics in a problem-centered, connected approach. Functions and the deductive methods of proof with geometric concepts are the principle topics of study. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Use the trigonometric ratios to model and solve problems.
- Apply geometric properties and relationships to solve problems.
- Use systems of linear equations or inequalities to model and solve problems.
- Define and use linear and exponential functions to model and solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Algebra

**COMPETENCY GOAL 1: The learner will perform operations with numbers and algebraic expressions to solve problems.**

### Objectives

- 1.01 Write equivalent forms of algebraic expressions to solve problems.
- 1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.
- 1.03 Simplify and perform operations with rational exponents and logarithms to solve problems.
- 1.04 Model and solve problems using direct, inverse, combined and joint variation.

**COMPETENCY GOAL 2: The learner will use properties of geometric figures to solve problems.**

**Objectives**

- 2.01 Use logic and deductive reasoning to draw conclusions and solve problems.
- 2.02 Apply the properties, definitions, and theorems of angles and lines to solve problems and write proofs.
- 2.03 Apply the properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs:
  - a) Triangles.
  - b) Quadrilaterals.
  - c) Other polygons.
  - d) Circles.
- 2.04 Use the law of sines and law of cosines to solve problems.

**COMPETENCY GOAL 3: The learner will use relations and functions to solve problems.**

**Objectives**

- 3.01 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.
- 3.02 Use quadratic functions and inequalities to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 3.03 Use rational equations to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
  - c) Identify the asymptotes and intercepts graphically and algebraically.
- 3.04 Use equations and inequalities with absolute value to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 3.05 Transform functions in two dimensions; describe the results algebraically and geometrically.

## Integrated Mathematics 4

Integrated Mathematics 4 provides students an advanced study of trigonometry, functions, analytic geometry, and data analysis with a problem-centered, connected approach in preparation for college-level mathematics. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

**Strands:** Number and Operations, Geometry and Measurement, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will operate with complex numbers and vectors to solve problems.**

### Objectives

- 1.01 Operate with vectors in two dimensions to model and solve problems.
- 1.02 Define and compute with complex numbers.

**COMPETENCY GOAL 2: The learner will describe geometric figures in the coordinate plane algebraically.**

**Objectives**

- 2.01 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 2.02 Estimate the area and volume of continuously varying quantities.

**COMPETENCY GOAL 3: The learner will analyze data to solve problems.**

**Objectives**

- 3.01 Analyze univariate data to solve problems.
  - a) Apply and compare methods of data collection.
  - b) Apply statistical principles and methods in sample surveys.
  - c) Determine measures of central tendency and spread.
  - d) Recognize, define, and use the normal distribution curve.
  - e) Interpret graphical displays of distributions of univariate data.
  - f) Compare distributions of univariate data.
- 3.02 Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions of bivariate data to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.

**COMPETENCY GOAL 4: The learner will use relations and functions to solve problems.**

**Objectives**

- 4.01 Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.
  - a) Solve using graphs and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.

- 4.02 Use recursively-defined functions to model and solve problems.
  - a) Find the sum of a finite sequence.
  - b) Find the sum of an infinite sequence.
  - c) Determine whether a given series converges or diverges.
  - d) Translate between recursive and explicit representations.
- 4.03 Use the composition and inverse of functions to model and solve problems.
- 4.04 Use trigonometric and inverse trigonometric functions to model and solve problems.
  - a) Solve using graphs and algebraic properties.
  - b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
- 4.05 Use polar equations to model and solve problems.
  - a) Solve using graphs and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 4.06 Use parametric equations to model and solve problems.
- 4.07 Find the rate of change at any point of a function.

## Advanced Placement Statistics

Advanced Placement Statistics introduces students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Students will observe patterns and departures from patterns, decide what and how to measure, produce models using probability and simulation, and confirm models. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Create and use, for sets of data, best-fit mathematical models of functions to solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use functions to model and solve problems.

**Strands:** Number and Operations, Data Analysis and Probability, Algebra

**COMPETENCY GOAL 1: The learner will analyze univariate data to solve problems.**

### Objectives

- 1.01 Summarize distributions of univariate data by determining and interpreting measures of center, spread, position, boxplots, and effects of changing units on summary measures.
- 1.02 Analyze distribution of continuous univariate data (both normal and non-normal).

**COMPETENCY GOAL 2: The learner will construct and interpret displays of univariate data to solve problems.**

### Objectives

- 2.01 Construct and interpret graphical displays of univariate data
- 2.02 Compare distributions among sets of univariate data.

**COMPETENCY GOAL 3: The learner will collect and analyze data to solve problems.**

**Objectives**

- 3.01 Analyze categorical data.
- 3.02 Use and compare methods of data collection.
- 3.03 Apply statistical principles and methods in sample surveys; identify difficulties.
- 3.04 Apply principles and methods in designed experiments; identify difficulties.
- 3.05 Apply concepts of probability to solve problems.
- 3.06 Use normal distributions as a model for distribution.
  - a) Investigate the properties of the normal distribution.
  - b) Use the table of standard normal distribution ( $Z$ ).
- 3.07 Simulate sampling distributions.
- 3.08 Use simulations to develop an understanding of the Central Limit Theorem and its importance in confidence intervals and tests of significance.
- 3.09 Recognize, construct and interpret results using confidence intervals in the context of a problem.
- 3.10 Perform tests of significance and interpret results in the context of a problem.

**COMPETENCY GOAL 4: The learner will analyze bivariate data to solve problems.**

**Objective**

- 4.01 Analyze bivariate data.
  - a) Recognize and analyze correlation and linearity.
  - b) Determine the least squares regression line.
  - c) Create residual plots and identify outliers and influential points to analyze data.
  - d) Use logarithmic and power transformations to analyze data.

## Advanced Placement Calculus

Advanced Placement Calculus develops the student's understanding of the concepts of calculus (functions, graphs, limits, derivatives and integrals) and provides experience with its methods and applications. The course encourages the geometric, numerical, analytical, and verbal expression of concepts, results, and problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

### Prerequisites

- Use circle, trigonometric, and inverse trigonometric functions to solve problems.
- Use the trigonometric ratios and the laws of sines and cosines to solve problems.
- Describe graphically, algebraically and verbally phenomena as functions; identifying independent and dependent quantities, domain, and range.
- Translate among graphic, algebraic, tabular, and verbal representations of relations.
- Use functions (linear, polynomial, exponential, logarithmic, rational, power, piecewise) to model and solve problems.
- Use the composition and inverse of functions to model and solve problems.
- Transform relations in two and three dimensions; describe algebraically and/or geometrically the results.
- Use the conic relations to model and solve problems.
- Write equivalent forms of algebraic expressions.
- Find special points (zeros, intercepts, asymptotes, local maximum, local minimum, etc.) of relations and describe in the context of the problem.

**Strands:** Number and Operations, Geometry and Measurement, Algebra

**COMPETENCY GOAL 1: The learner will demonstrate an understanding of the behavior of functions.**

### Objectives

- 1.01 Demonstrate an understanding of limits both local and global.
  - a) Calculate limits, including one-sided, using algebra.
  - b) Estimate limits from graphs or tables of data.

- 1.02 Recognize and describe the nature of aberrant behavior caused by asymptotes and unboundedness.
  - a) Understand asymptotes in terms of graphical behavior.
  - b) Describe asymptotic behavior in terms of limits involving infinity.
  - c) Compare relative magnitudes of functions and their rates of change.
- 1.03 Identify and demonstrate an understanding of continuity of functions.
  - a) Develop an intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
  - b) Understand continuity in terms of limits.
  - c) Develop a geometric understanding of graphs of continuous functions. (Intermediate Value Theorem and Extreme Value Theorem).

**COMPETENCY GOAL 2: The learner will use derivatives to solve problems.**

**Objectives**

- 2.01 Explore and interpret the concept of the derivative graphically, numerically, analytically and verbally.
  - a) Interpret derivative as an instantaneous rate of change.
  - b) Define derivative as the limit of the difference quotient.
  - c) Identify the relationship between differentiability and continuity.
- 2.02 Apply the concept of the derivative at a point.
  - a) Find the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
  - b) Find the tangent line to a curve at a point and local linear approximation.
  - c) Find the instantaneous rate of change as the limit of average rate of change.
  - d) Approximate a rate of change from graphs and tables of values.
- 2.03 Interpret the derivative as a function.
  - a) Identify corresponding characteristics of graphs of  $f$  and  $f'$ .
  - b) Identify relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$ .
  - c) Investigate the Mean Value Theorem and its geometric consequences.
  - d) Translate between verbal and algebraic descriptions of equations involving derivatives.

- 2.04 Demonstrate fluency and accuracy in the computation of derivatives.
- Find the derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
  - Use the basic rules for the derivative of sums, products, and quotients of functions.
  - Use the chain rule and implicit differentiation.
- 2.05 Interpret the second derivative.
- Identify the corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$ .
  - Identify the relationship between the concavity of  $f$  and the sign of  $f''$ .
  - Identify points of inflection as places where concavity changes.
- 2.06 Apply the derivative in graphing and modeling contexts.
- Analyze curves, with attention to monotonicity and concavity.
  - Optimize with both absolute (global) and relative (local) extrema.
  - Model rates of change, including related rates problems.
  - Use implicit differentiation to find the derivative of an inverse function.
  - Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
  - Interpret differential equations geometrically via slope fields and the relationship between slope fields and solution curves for differential equations.

### COMPETENCY GOAL 3: The learner will use integrals to solve problems.

#### Objectives

- 3.01 Explore and interpret the concept of the definite integral.
- Compute Riemann sums using left, right, and midpoint evaluation points.
  - Find the definite integral as a limit of Riemann sums over equal subdivisions.
  - Find the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
 
$$\int_a^b f'(x)dx = f(b) - f(a)$$
  - Identify basic properties of definite integrals.

- 3.02 Apply standard techniques of anti-differentiation.
- Find anti-derivatives following directly from derivatives of basic functions.
  - Find anti-derivatives by substitution of variables. (including change of limits for definite integrals).
- 3.03 Apply and interpret the Fundamental Theorem of Calculus.
- Use the Fundamental Theorem to evaluate definite integrals.
  - Use the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined.
- 3.04 Define and use appropriate integrals in a variety of applications.
- Interpret the integral of a rate of change to give accumulated change.
  - Find specific anti-derivatives using initial conditions.
  - Set up and use an approximating Riemann sum or trapezoidal sum and represent its limit as a definite integral.
  - Find the area of a region.
  - Find the volume of a solid with known cross sections.
  - Find the average value of a function.
  - Find the distance traveled by a particle along a line.
  - Solve separable differential equations and use them in modeling. In particular, study the equation  $y' = ky$  and exponential growth.

